

ICTP – Trieste 2024



Modelling Optical Properties using WIEN2k



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Université
de Rennes

Modelling the Optical Properties of Inorganic Materials

1 – MULTIPLE FACETS OF COLOURED MATTER

2 – ELECTRONIC STRUCTURE OF A SOLID

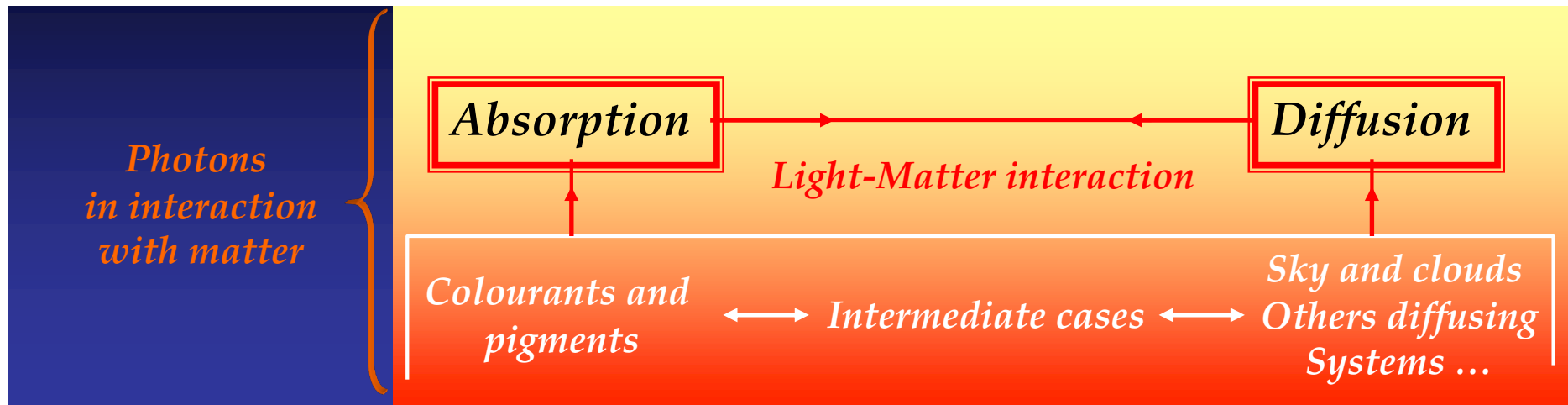
3 – UNDERSTANDING OF COLORS FROM BANDS

4 – LIGHT-MATTER INTERACTION

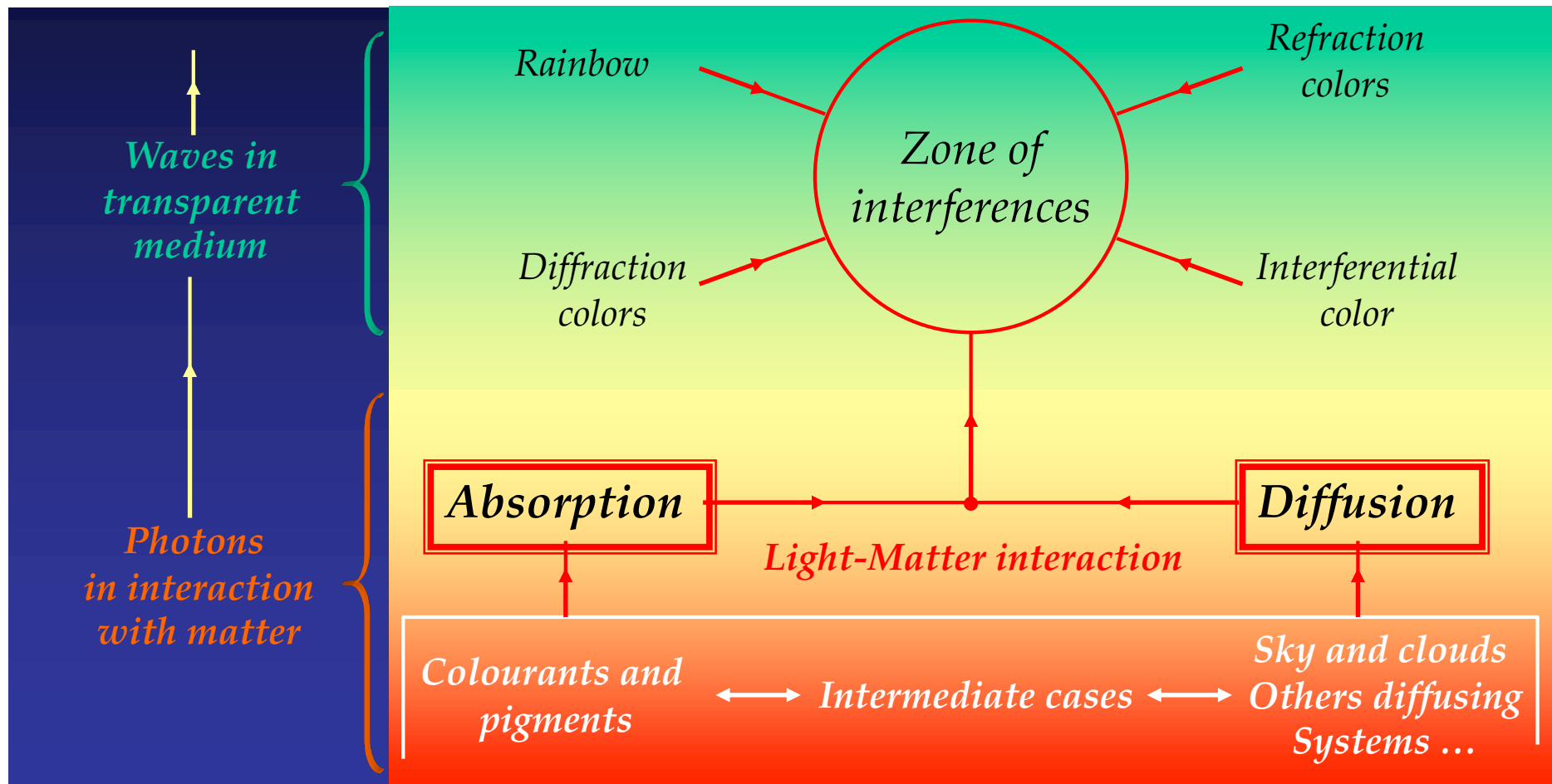
5 – OPTICAL PROPERTIES: WHICH TREATMENT?

6 – ILLUSTRATIONS

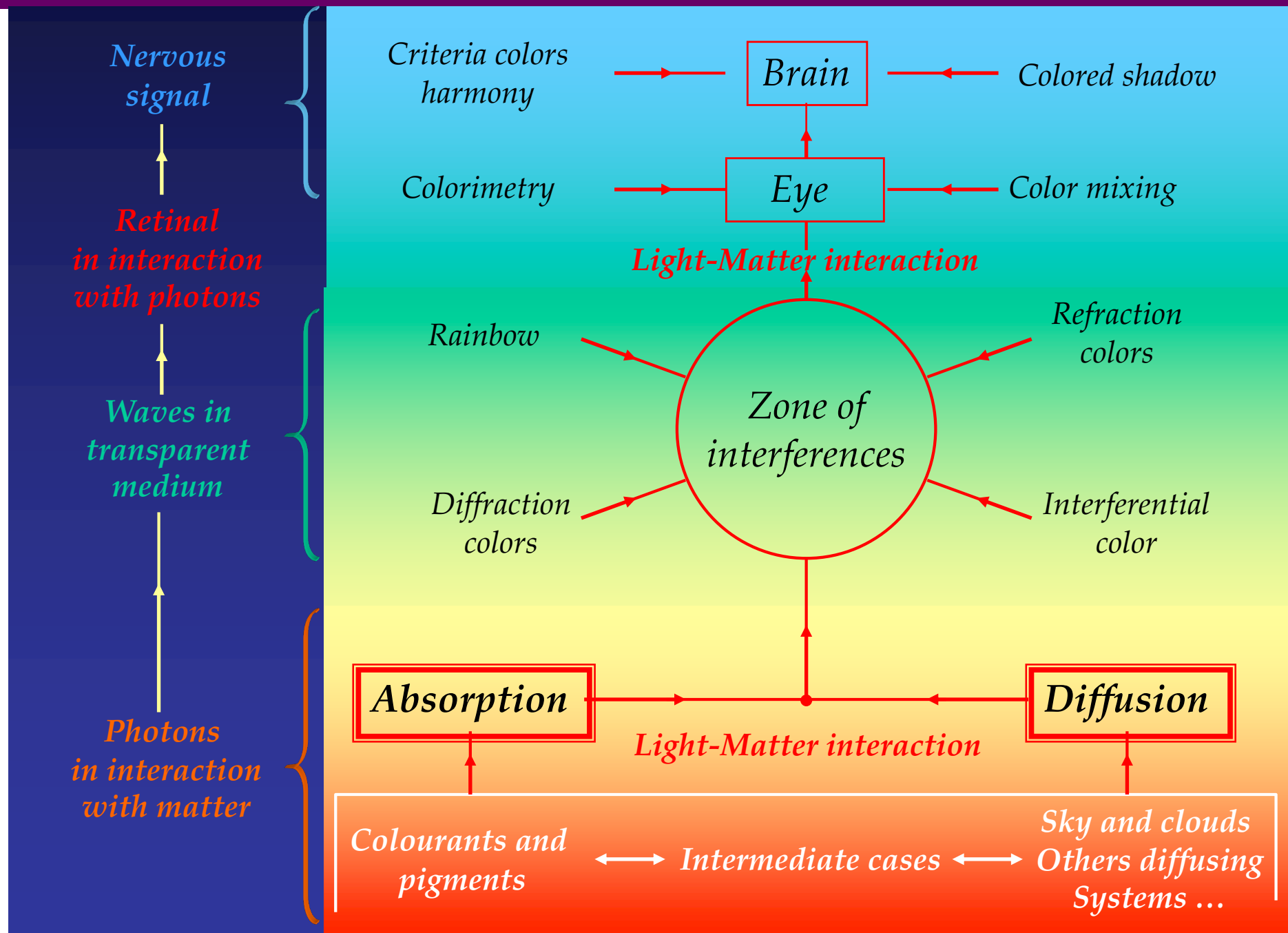
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Interaction between the electromagnetic field and matter

Physical color



Diffusion et interferences

Dispersion / Reflection et Refraction / Scattering

Transparent matter

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Interaction between the electromagnetic field and matter

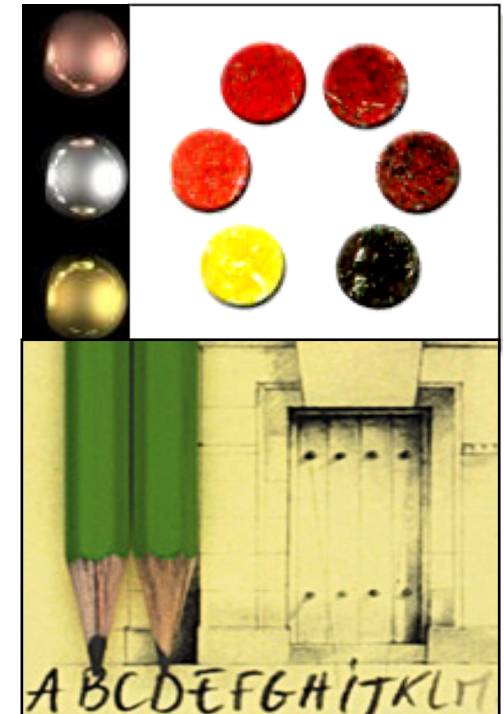
Physical color



*Diffusion et interferences
Dispersion / Reflection et Refraction / Scattering*

Transparent matter

Chemical color



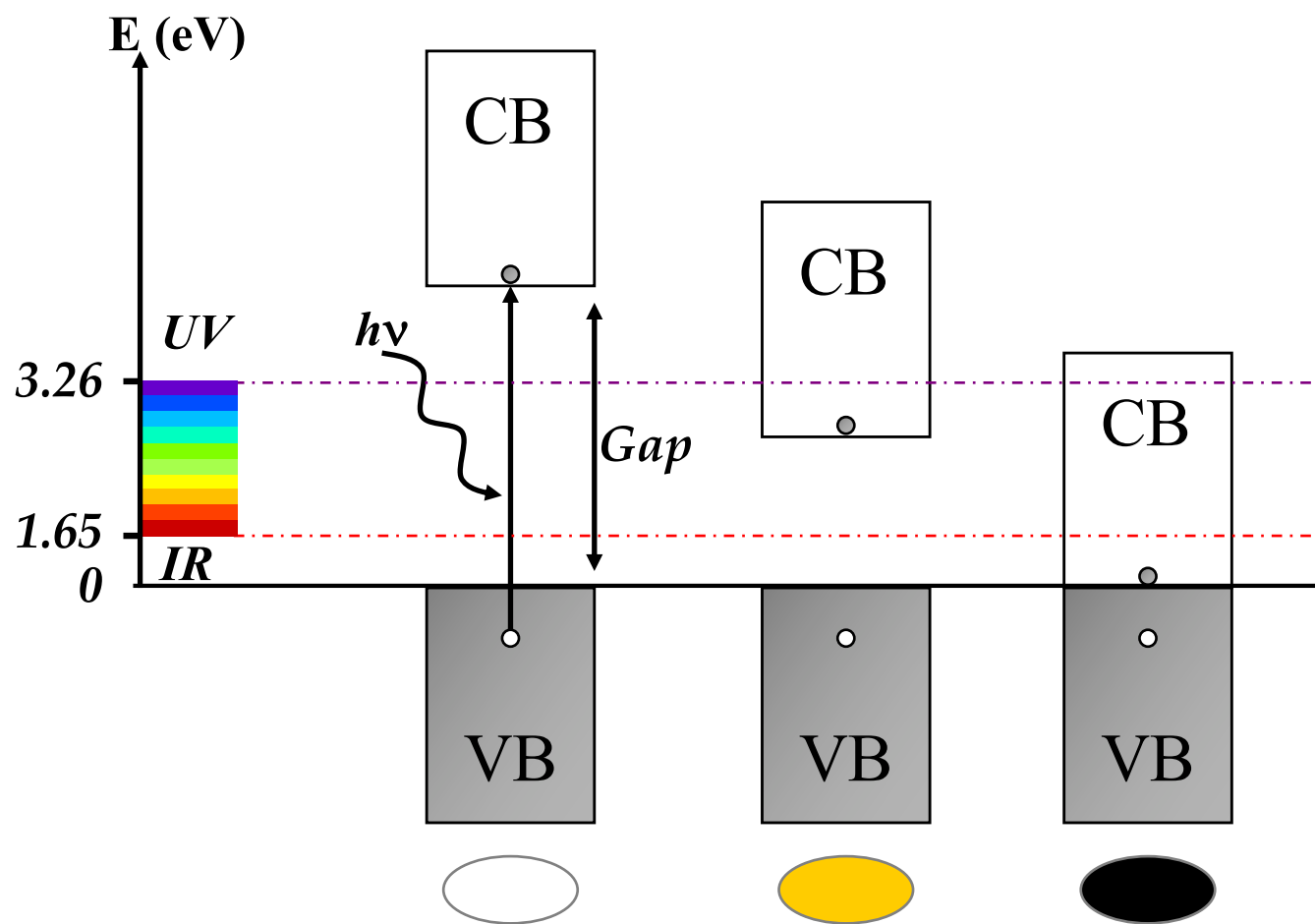
*Absorption
Energy dissipation
Opaque matter*

1 – MULTIPLE FACETS OF COLOURED MATTER

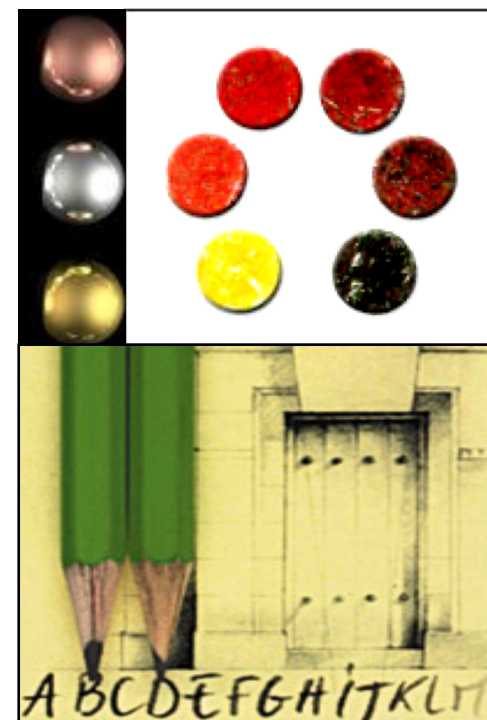
Chemical color → « *Inelastic diffusion* »

Acceptor electron levels ⇒ *Dissipative absorption*

From insulator to semiconductor to metal systems



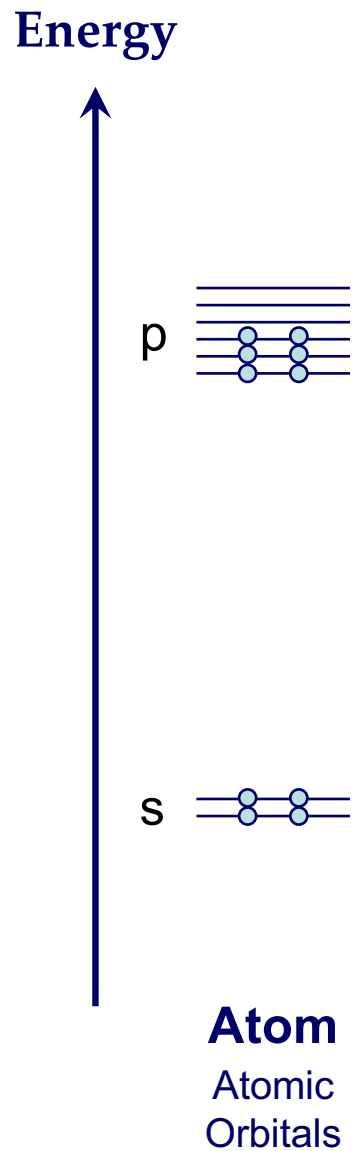
Chemical colors



Absorption
Energy dissipation
Opaque Matter

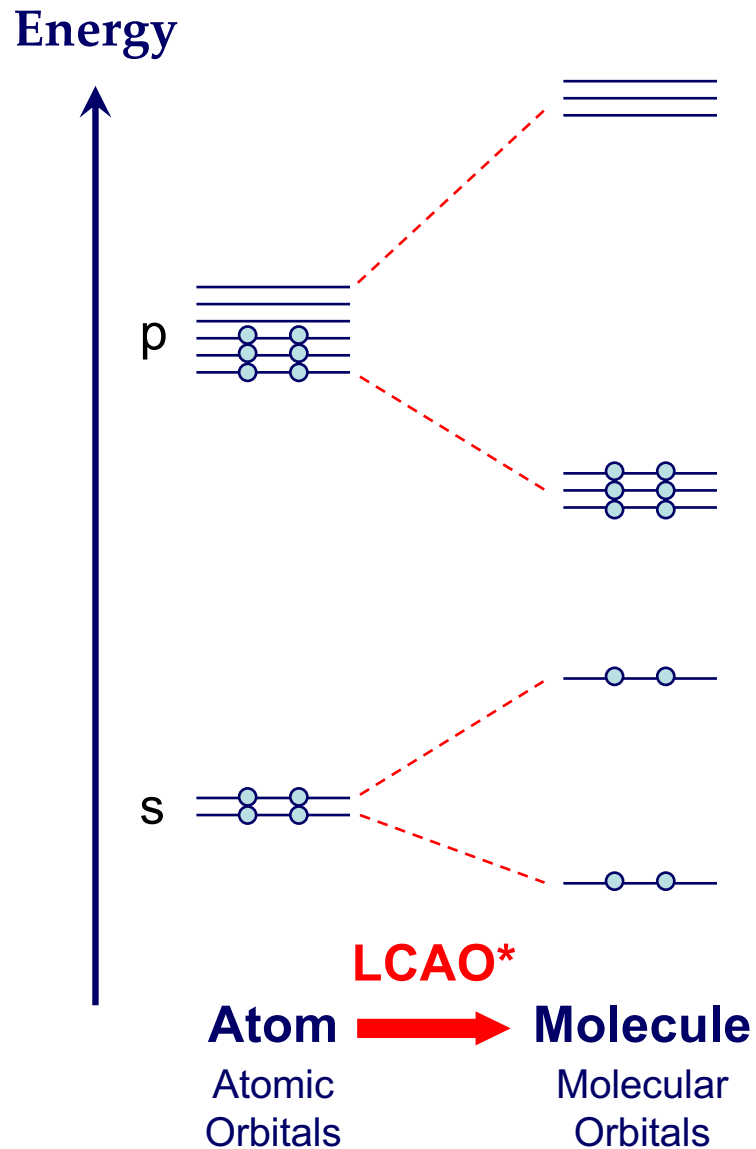
2 – ELECTRONIC STRUCTURE OF A SOLID

From the atom to the molecule and to the solid



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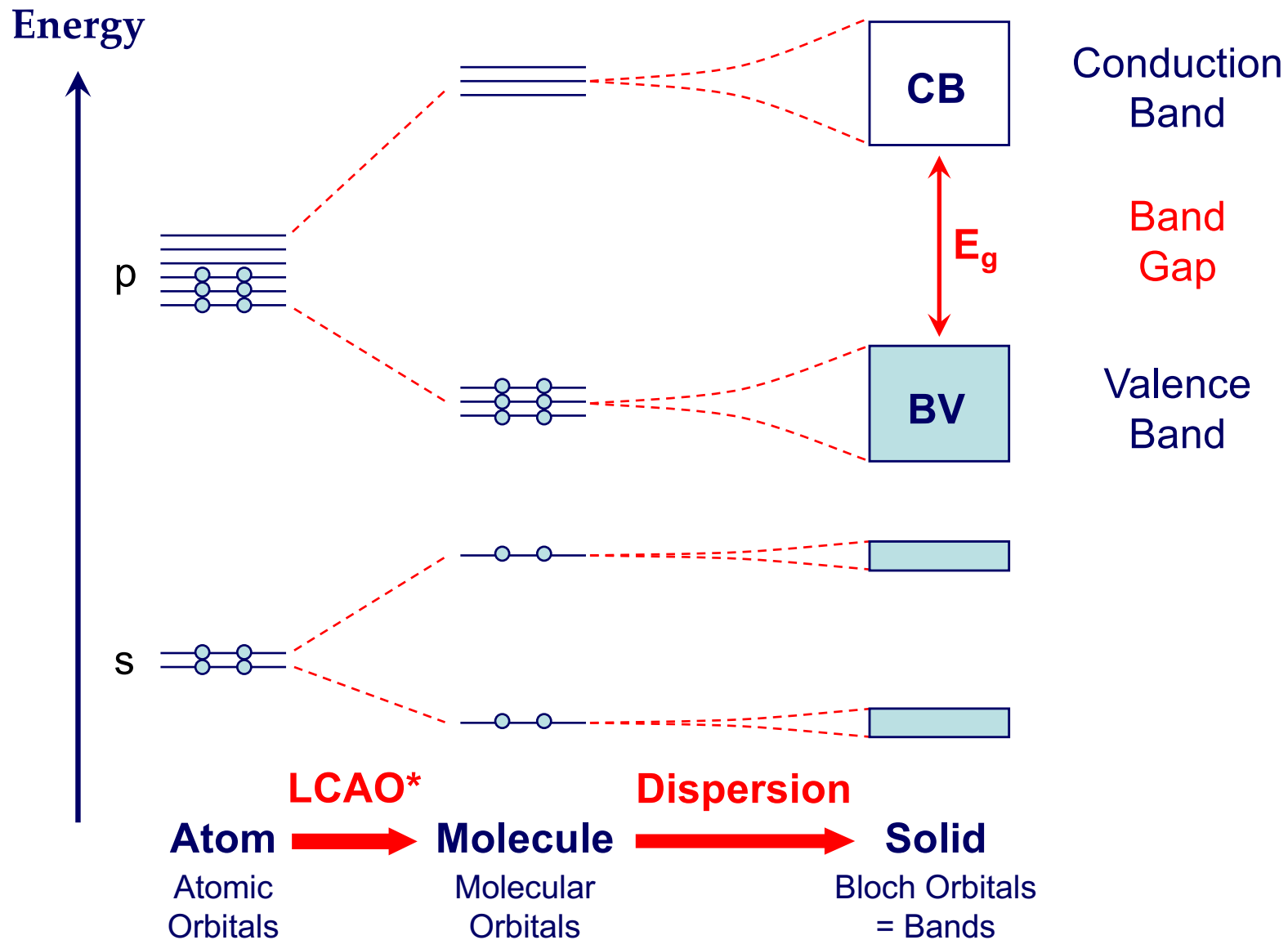
From the atom to the molecule and to the solid



*LCAO : Linear Combination of Atomic Orbitals

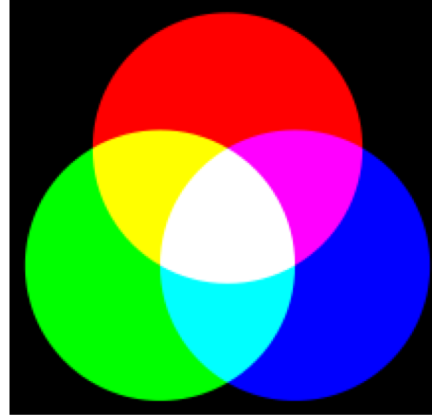
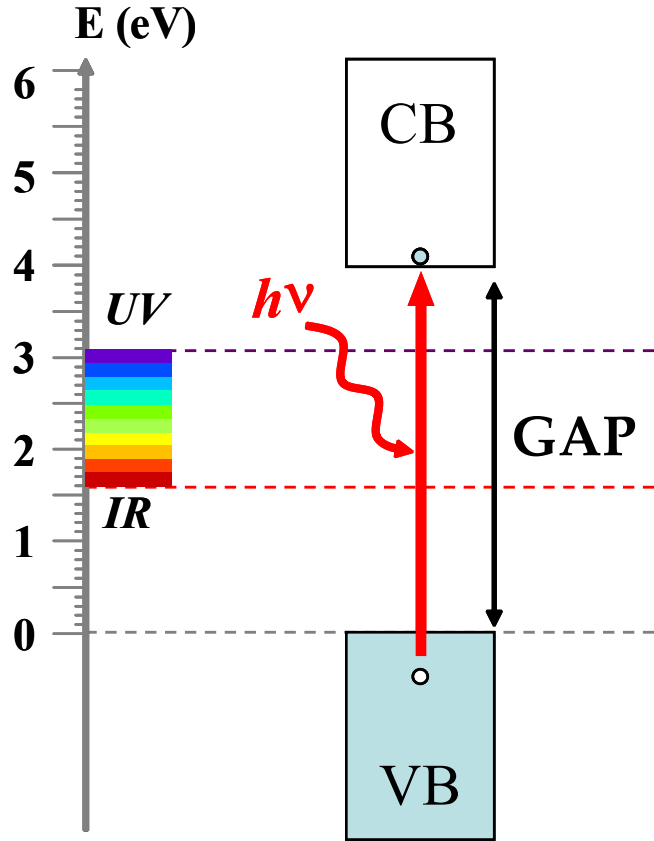
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From the atom to the molecule and to the solid

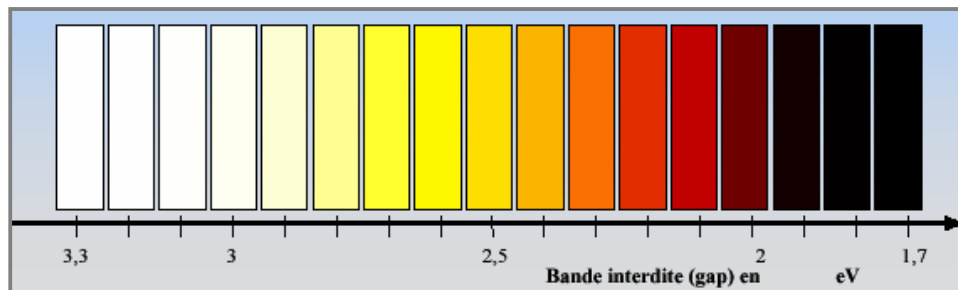


*LCAO : Linear Combination of Atomic Orbitals

3 – UNDERSTANDING OF COLORS FROM BANDS

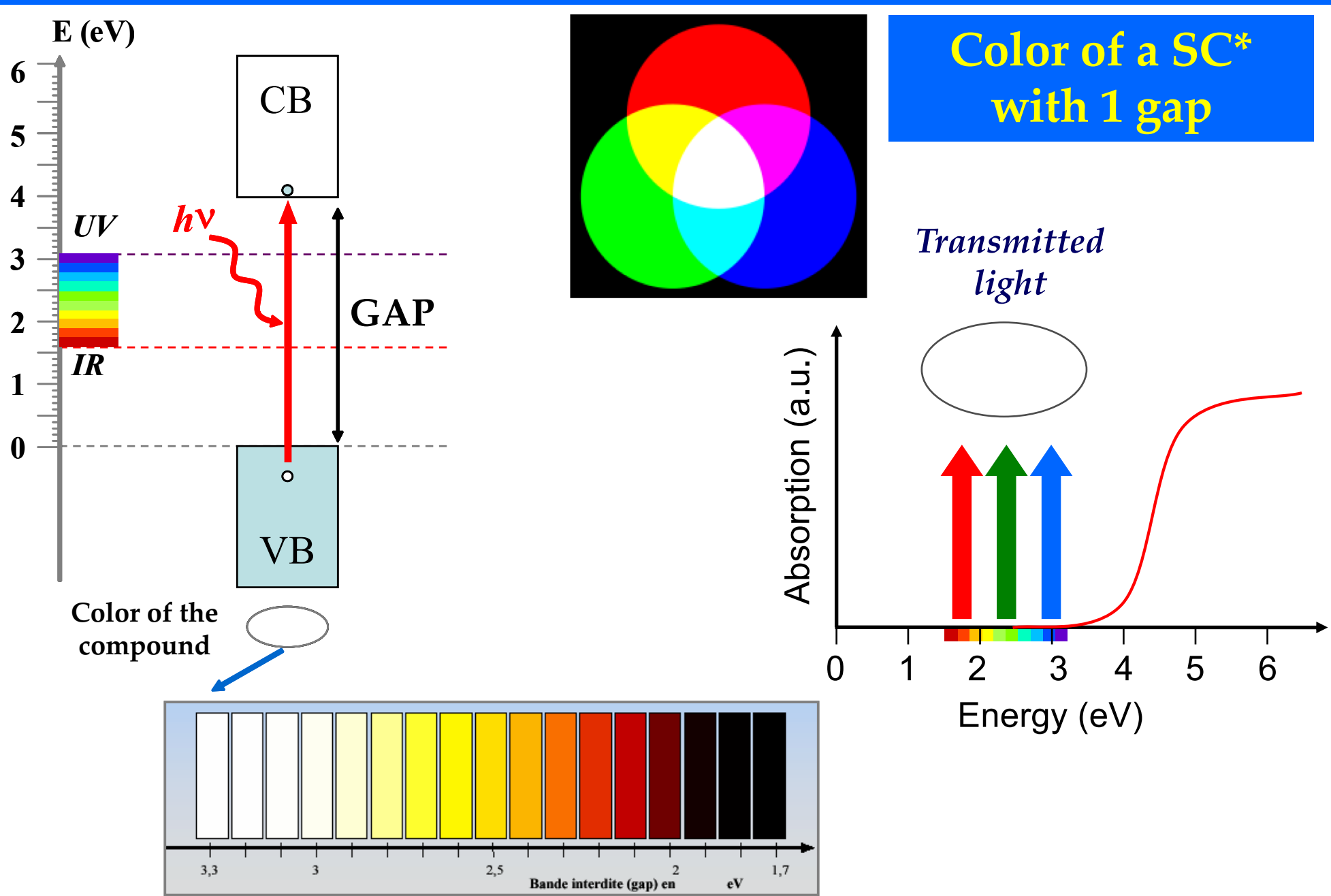


Color of a SC* with 1 gap



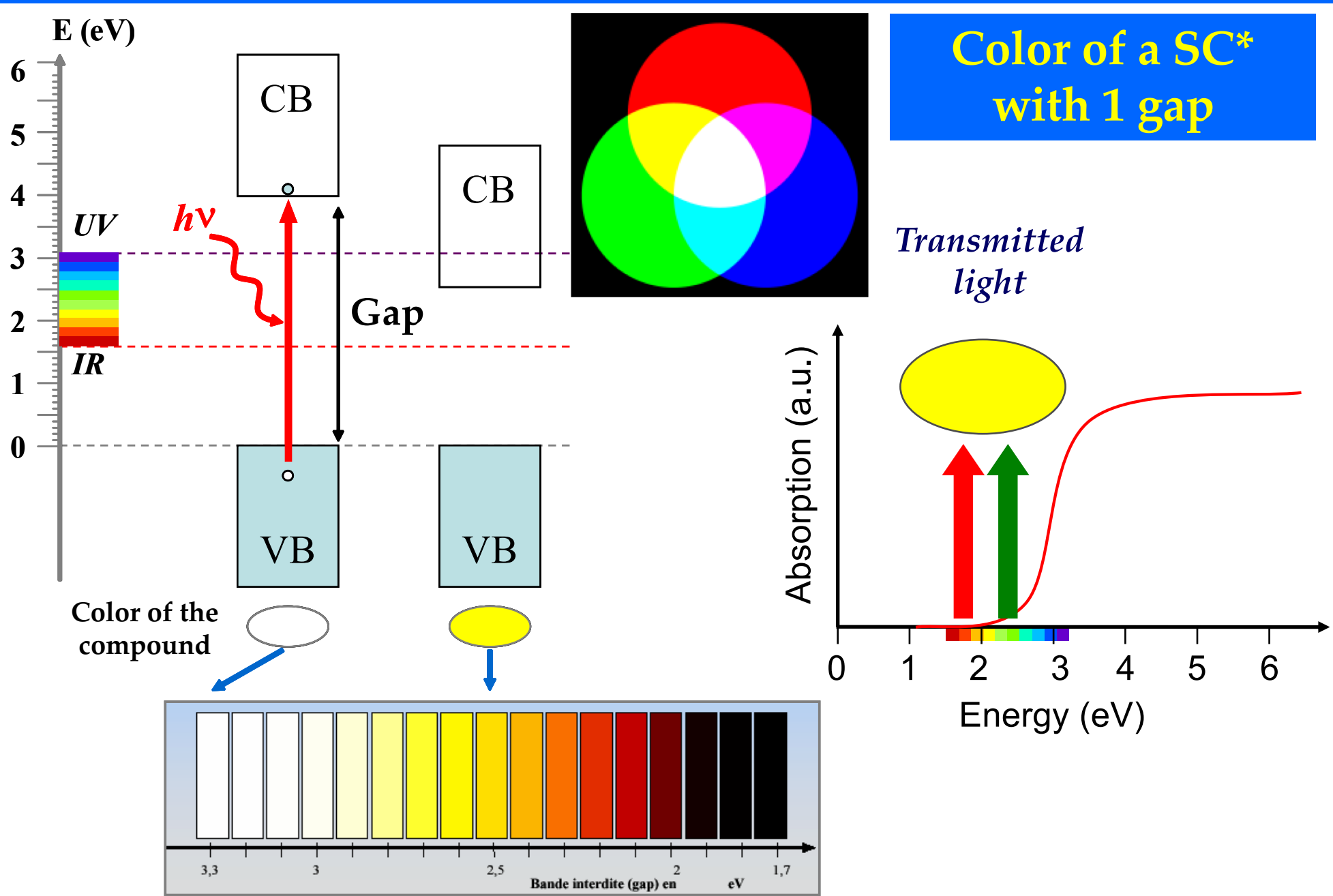
*SC : Semiconductor

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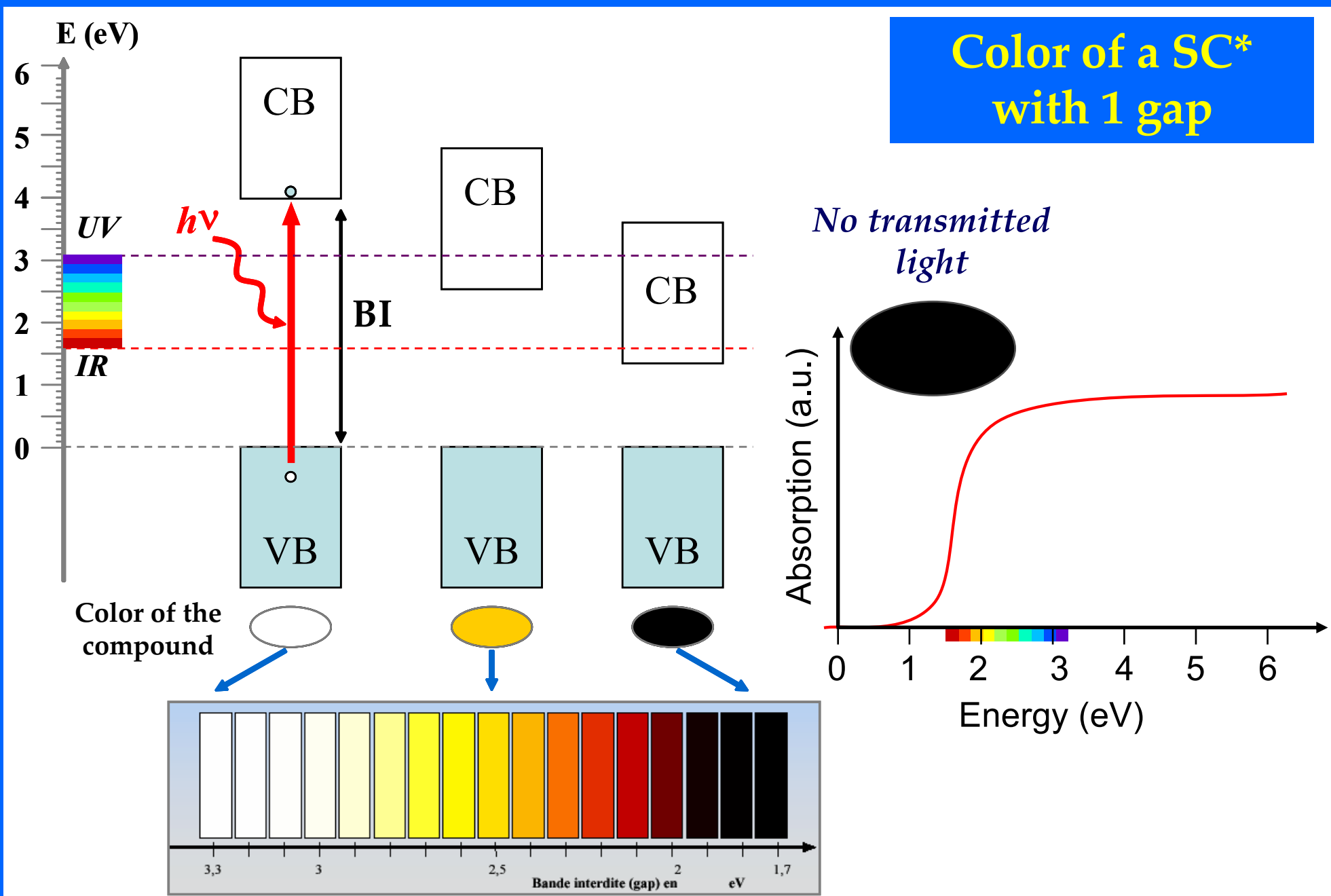
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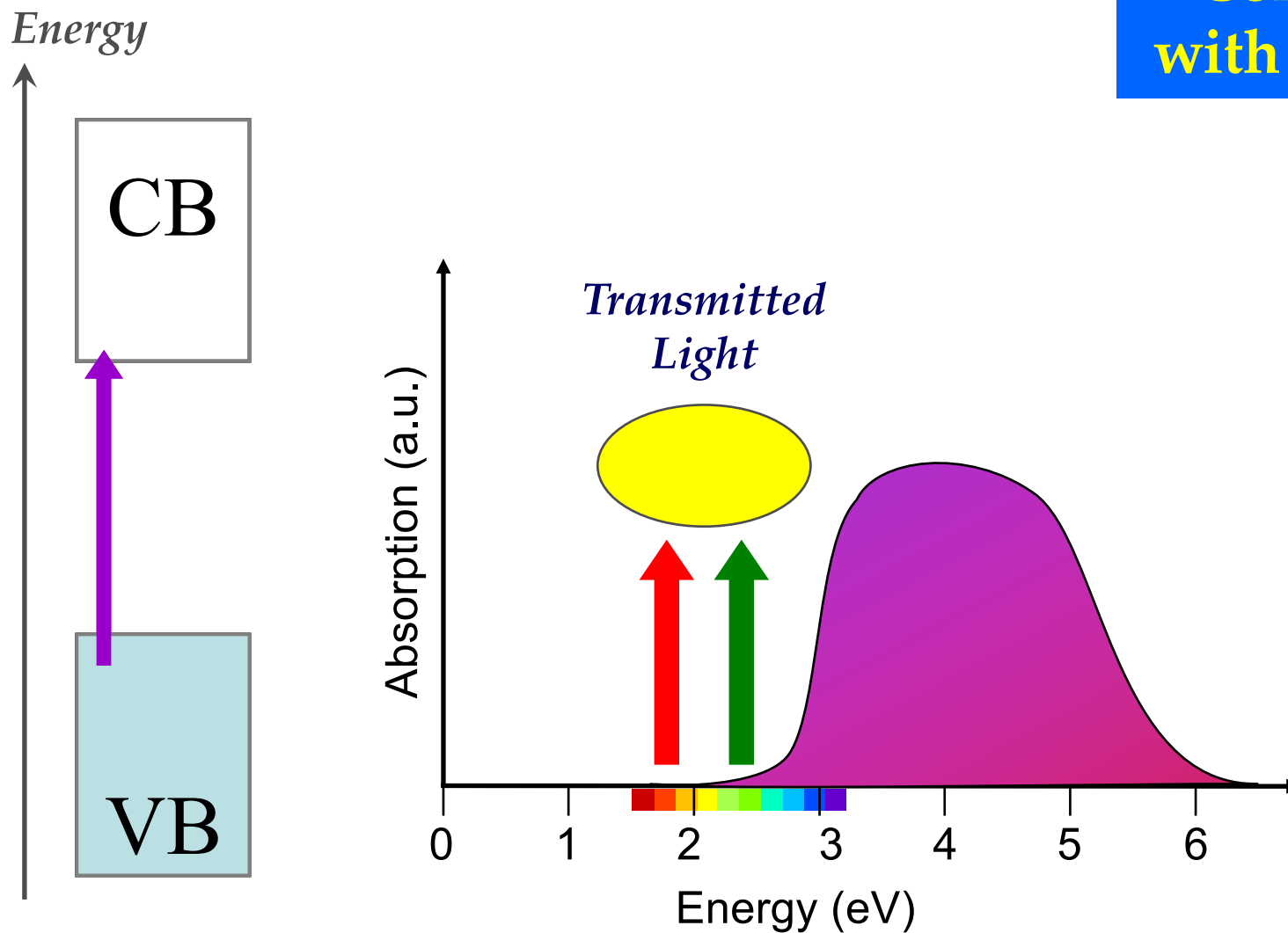
Color of a SC* with 1 gap

No transmitted light

*SC : Semiconductor

3 – UNDERSTANDING OF COLORS FROM BANDS

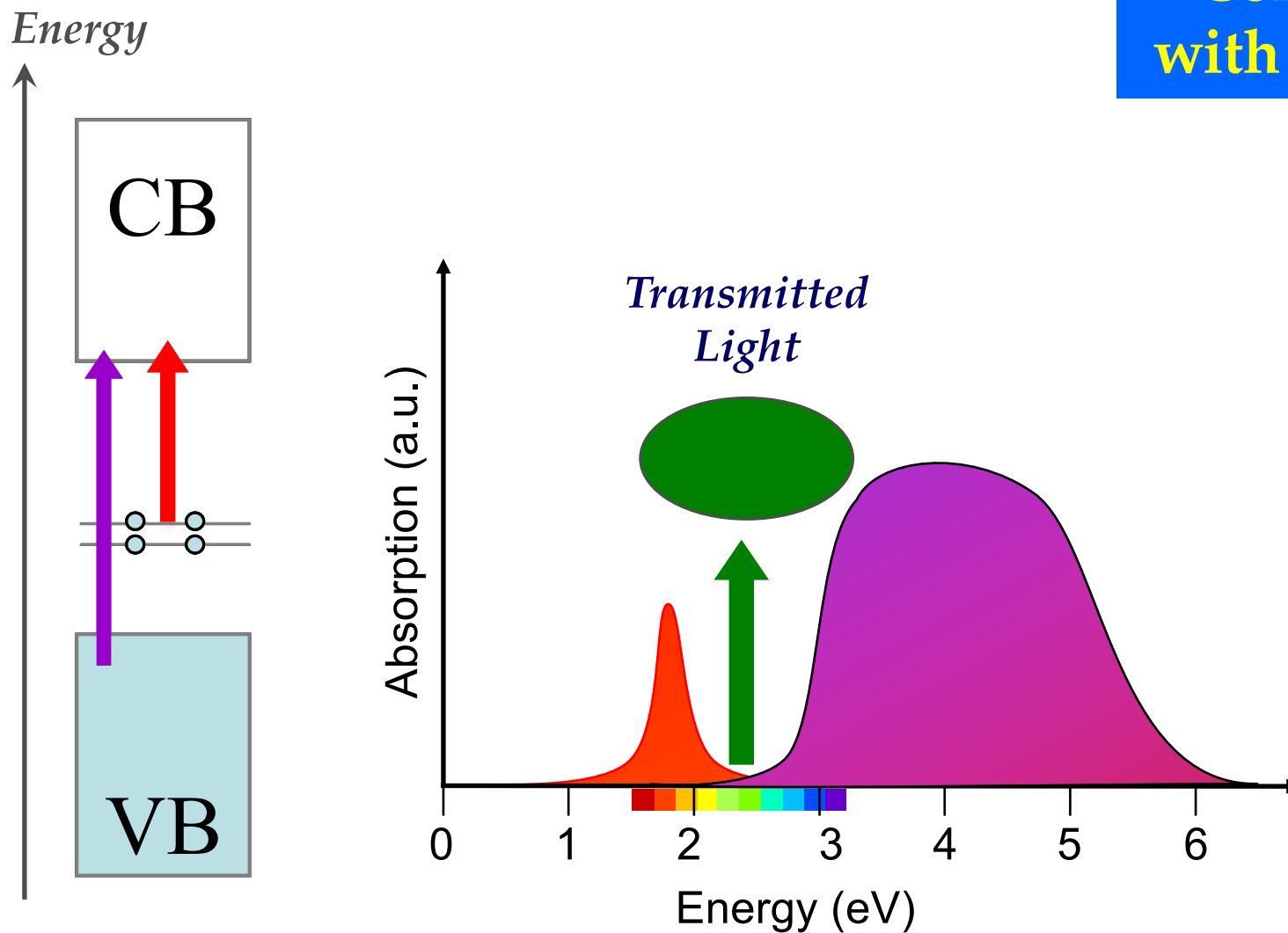
Color of a SC*
with several gaps



*SC : Semiconductor

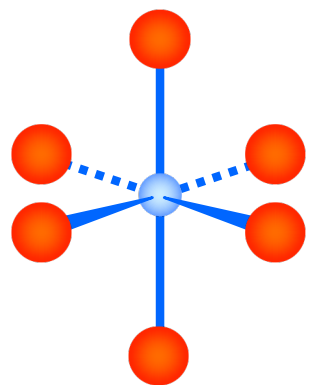
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Color of a SC*
with several gaps



*SC : Semi-conducteur – BI : Bande interdite

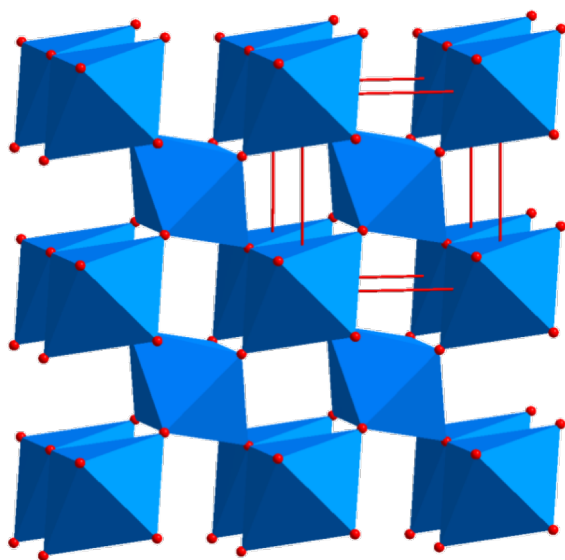
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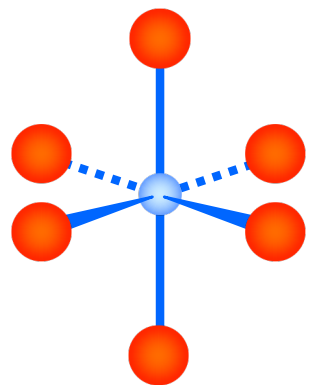
*Powder sample
of TiO₂ (rutile)*



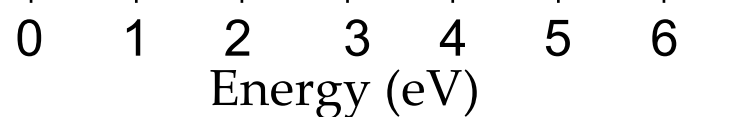
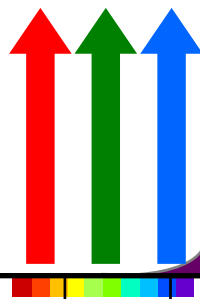
**Charge transfer →
color**



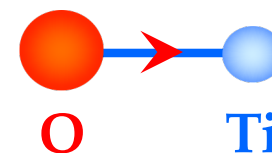
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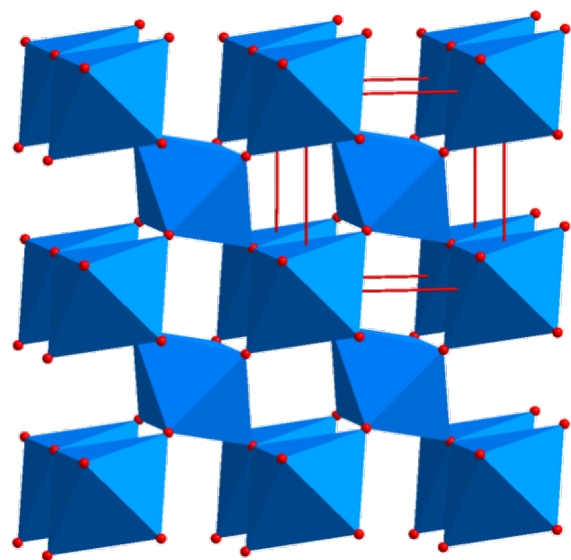
Absorption (a.u.)



Charge transfer → color

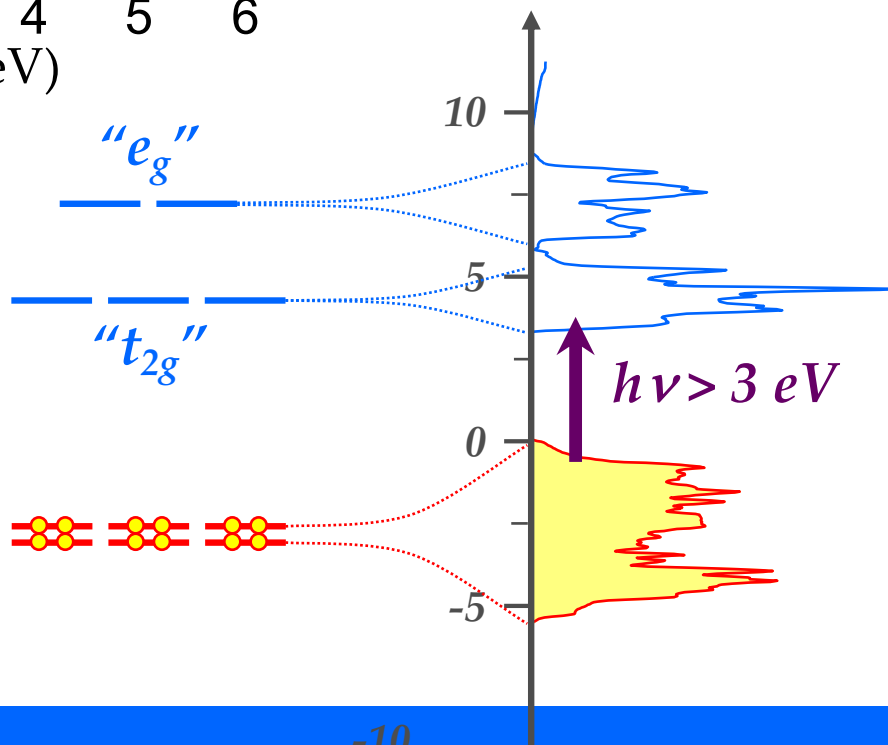


Energy (eV)

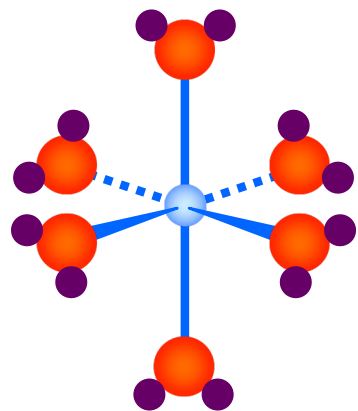


$Ti^{4+} (3d^0)$

$2 \times O^{2-} (2p^6)$



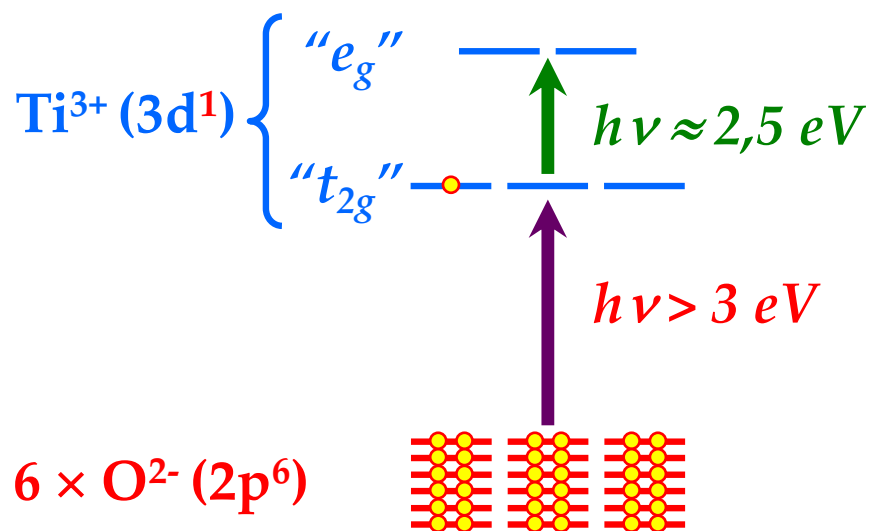
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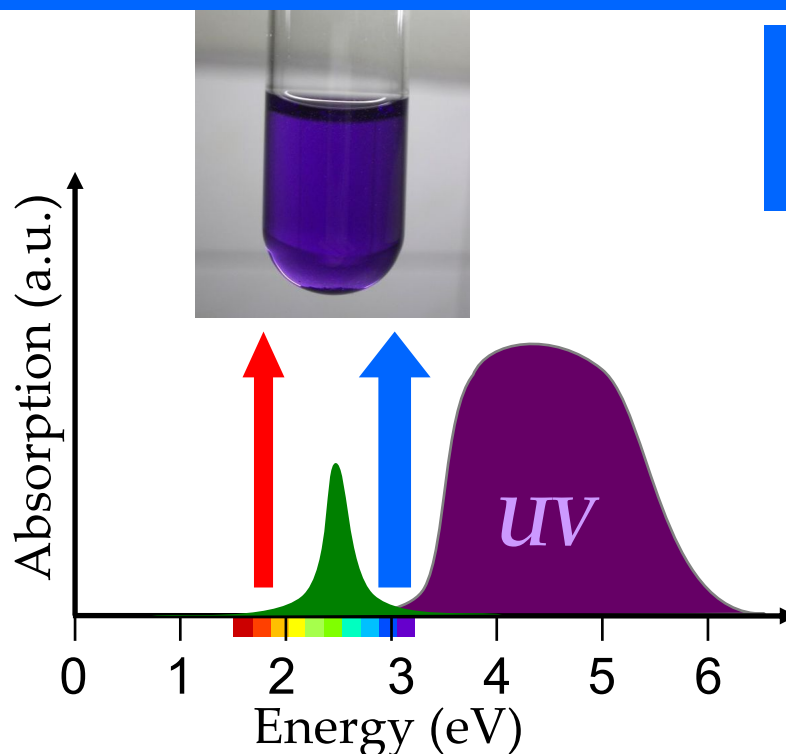
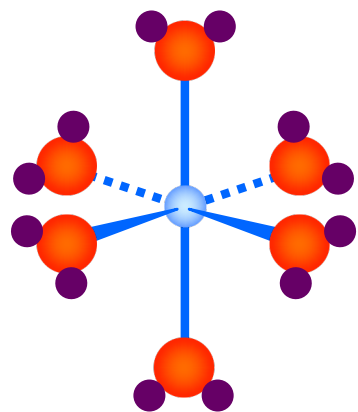
$TiCl_3$
in solution



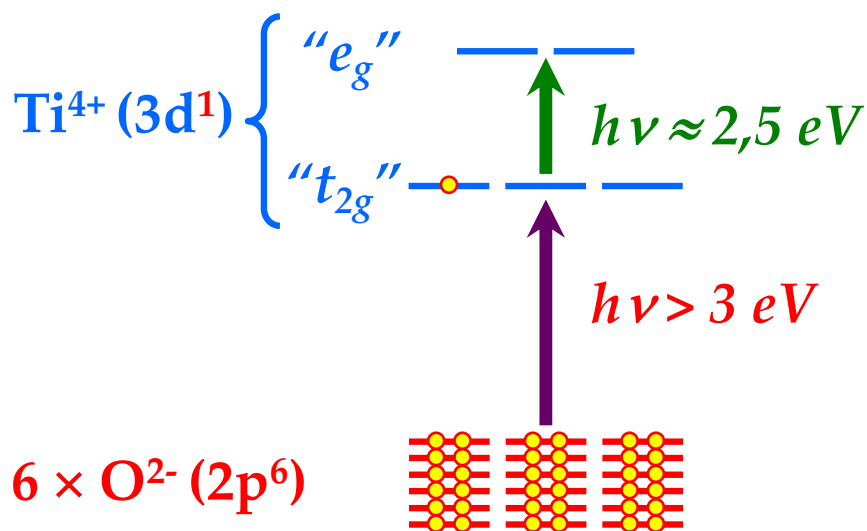
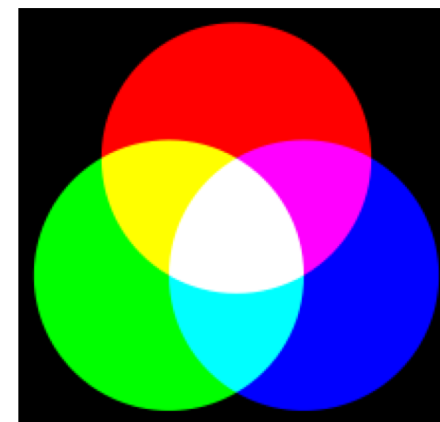
d-d transition
→ color



3 – UNDERSTANDING OF COLORS FROM BANDS



d-d transition
→ color

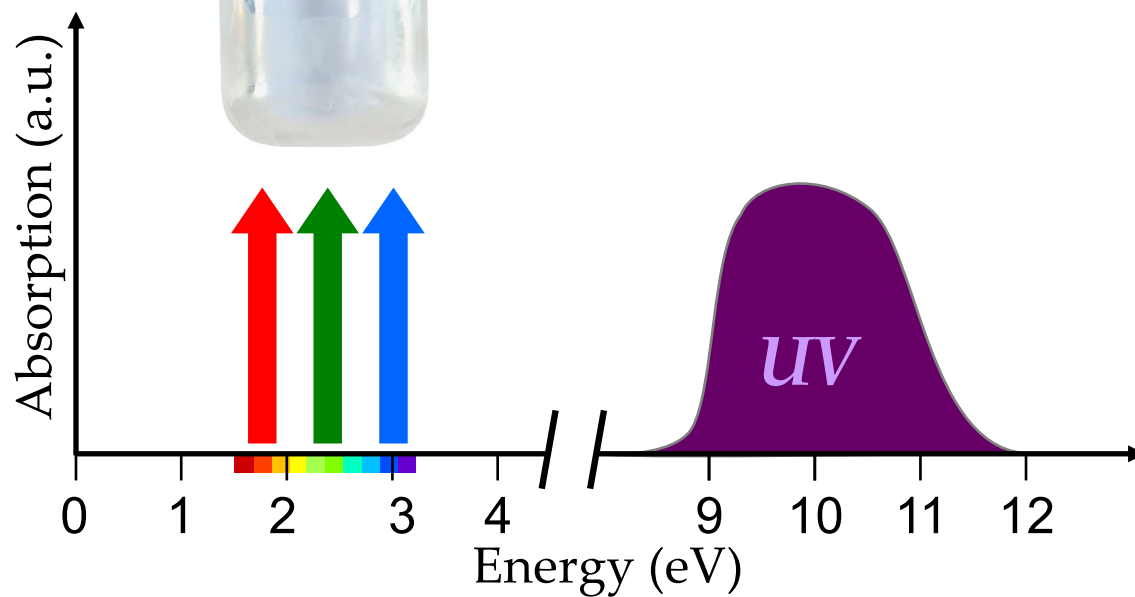


Rq. : For d^n with $n > 1$

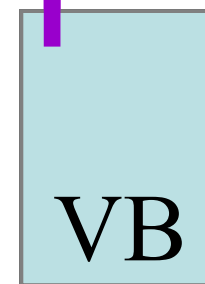
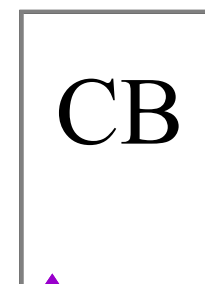
- Interelectronic correlations must be taken into account
- Tanabe-Sugano diagrams (spectroscopic terms)

3 – UNDERSTANDING OF COLORS FROM BANDS

Alumine
corindon
(Al_2O_3)



Energie



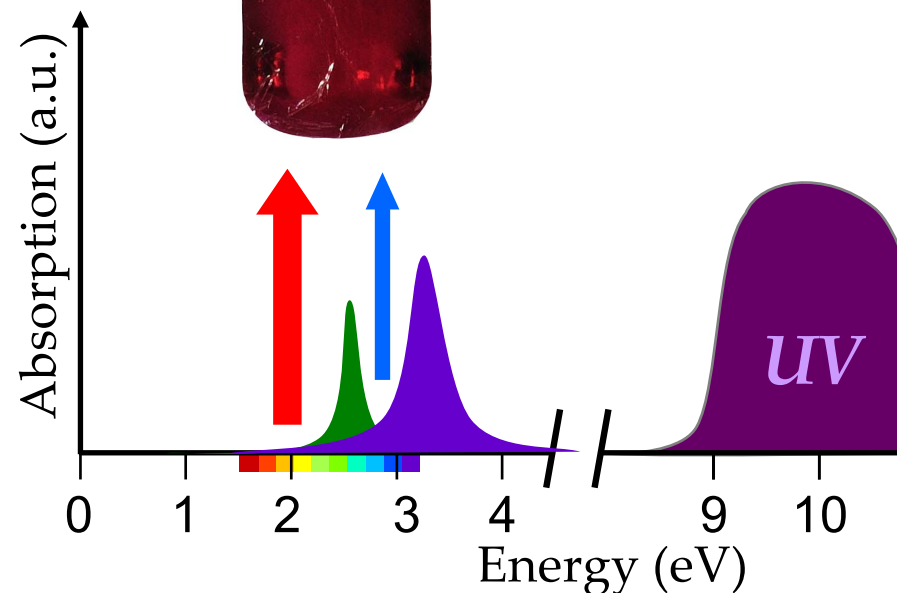
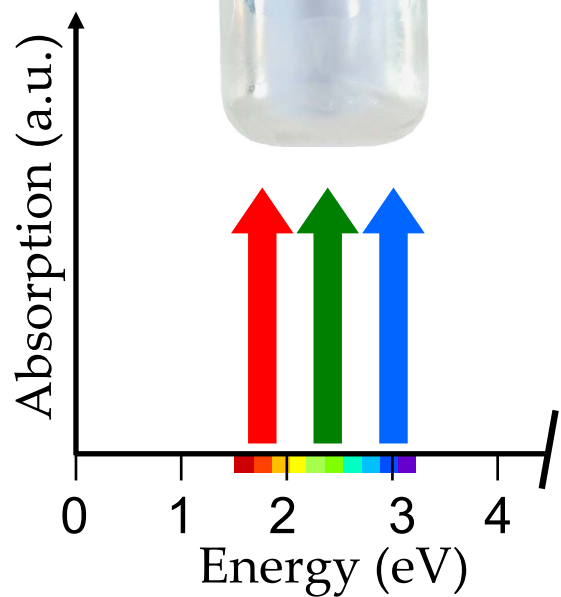
3 – UNDERSTANDING OF COLORS FROM BANDS

*Alumina
corindon
(Al_2O_3)*



+ 1% Cr^{3+}

*Synthetic
rubis
($Al_2O_3:1\%Cr^{3+}$)*



3 – UNDERSTANDING OF COLORS FROM BANDS

Rubis
($\text{Al}_2\text{O}_3:1\%\text{Cr}^{3+}$)

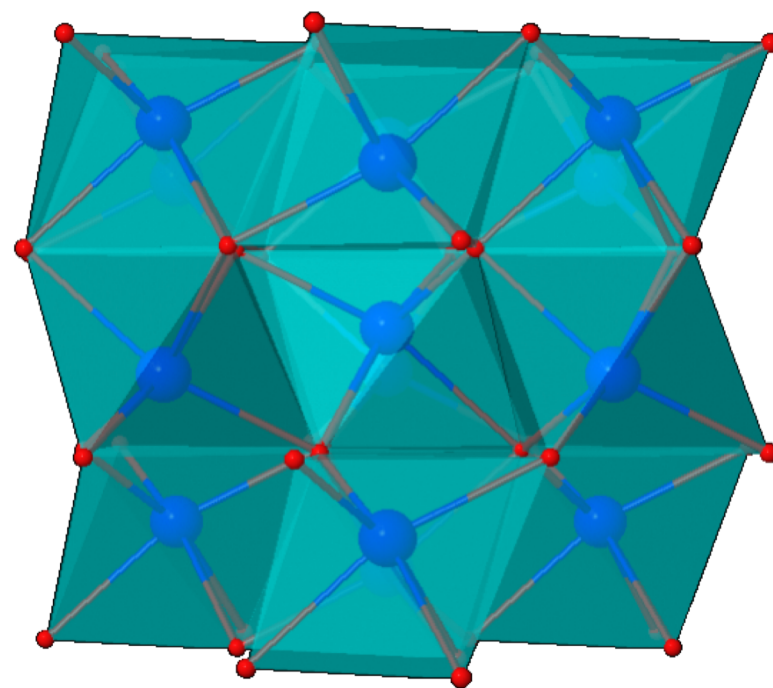


Impurity (d-d)
→ color

Cr^{3+} : electronic conf. d^3

→ Interelectronic correlations must be taken into account

→ Tanabe-Sugano diagrams (spectroscopic terms)



3 – UNDERSTANDING OF COLORS FROM BANDS

Rubis
($Al_2O_3:1\%Cr^{3+}$)

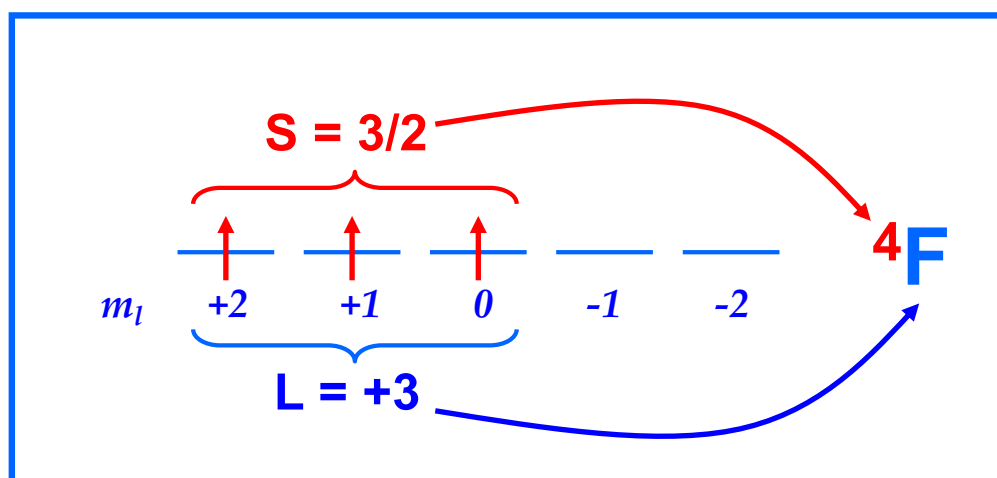
Impurity (d-d)
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Cr^{3+} : elect. conf. d^3



Free ion: we define the spectroscopic terms :

Spin multiplicity → $2S+1$ X



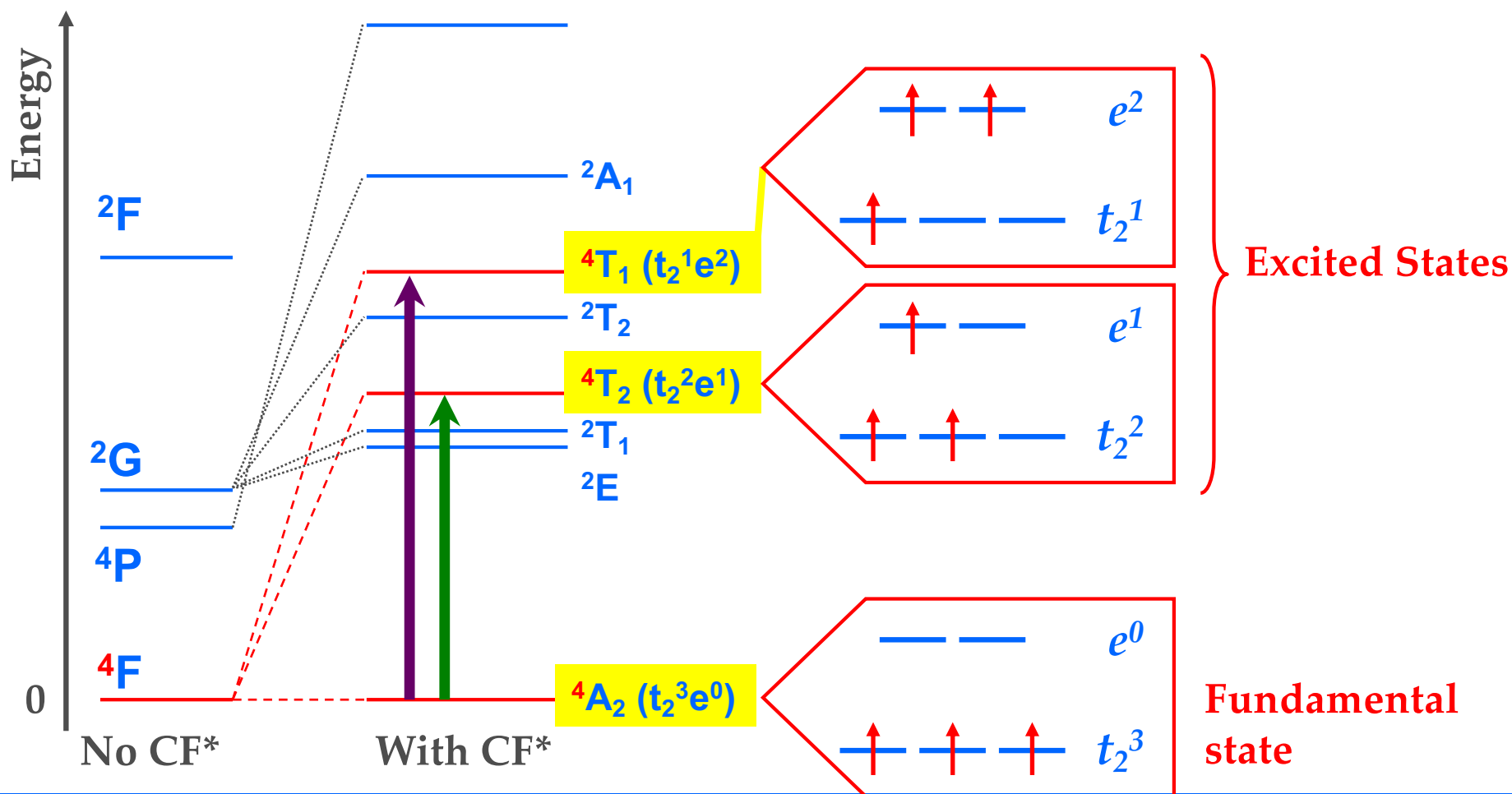
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Rubis
($Al_2O_3:1\%Cr^{3+}$)

Impurity (d-d)
→ color

Cr^{3+} : conf. $e^g d^3$

→ Degeneracy lifting due to the octahedral crystal field effect created by the ligands



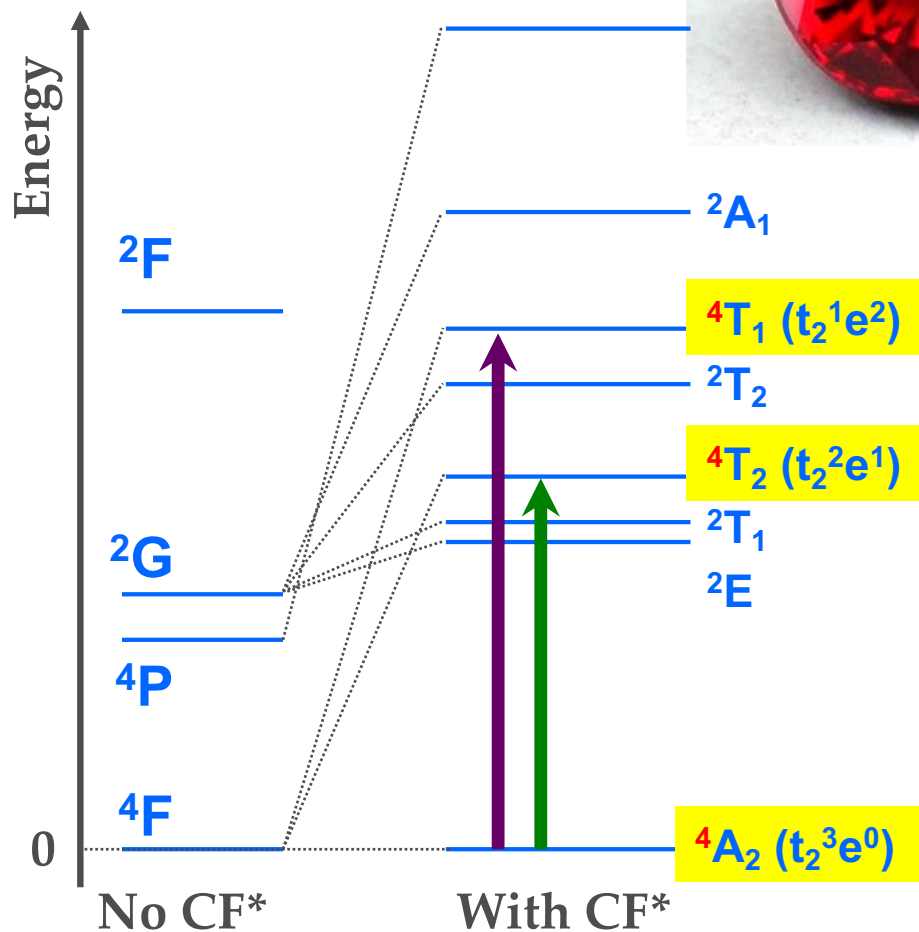
*CF : Cristal Field

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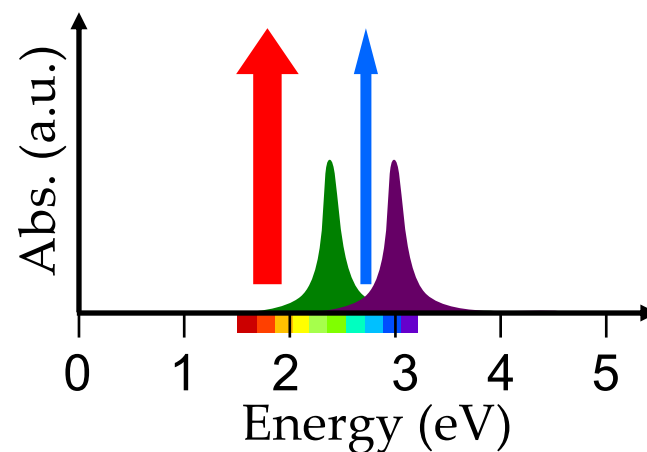
Rubis
($Al_2O_3:1\%Cr^{3+}$)



Impurity (d-d)
→ color



Red color
+
Slightly blue



Cr^{3+} : electronic conf. d^3

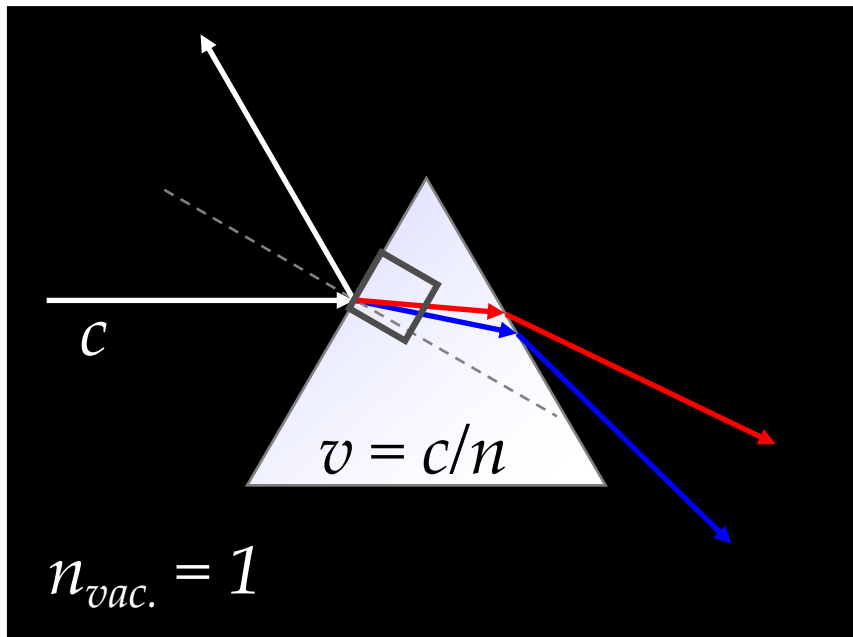
- Interelectronic correlations must be taken into account
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*CF : Cristal Field

4 – LIGHT-MATTER INTERACTION

Physical color → « Elastic diffusion »

Example of a prism



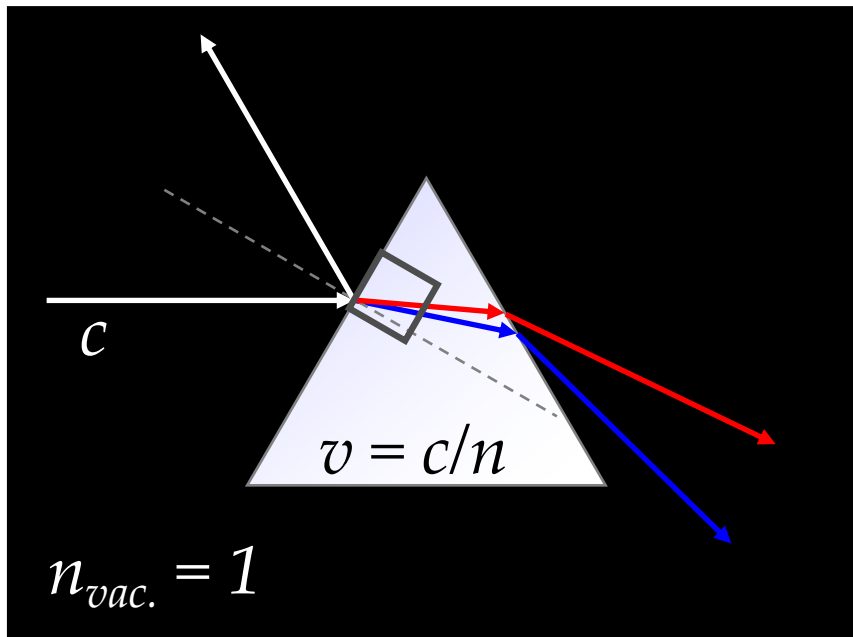
n : index of refraction

Propagation speed is changed

4 – LIGHT-MATTER INTERACTION

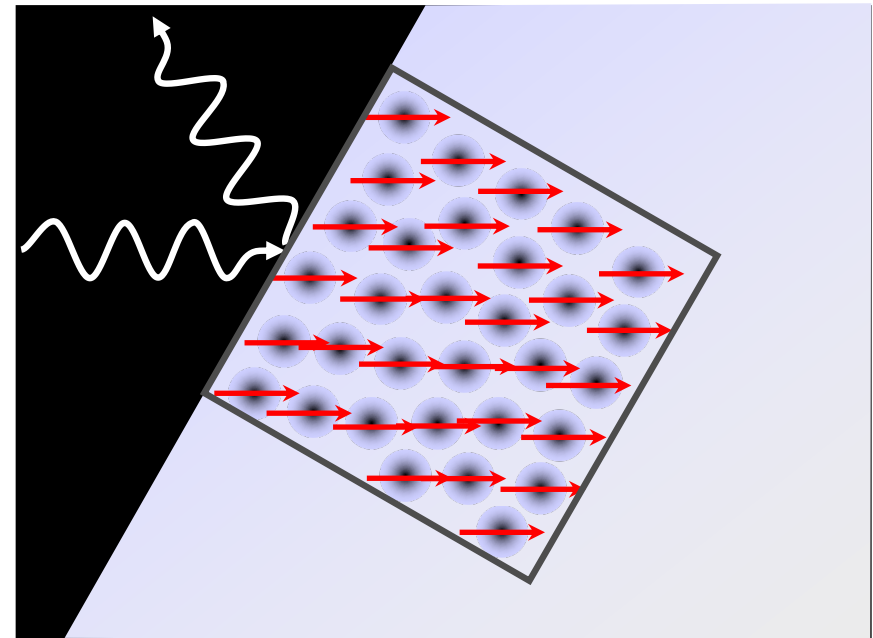
Physical color → « Elastic diffusion »

Example of a prism



n : index of refraction
Propagation speed is changed

*Elastic diffusion mechanism
at the atomic level*

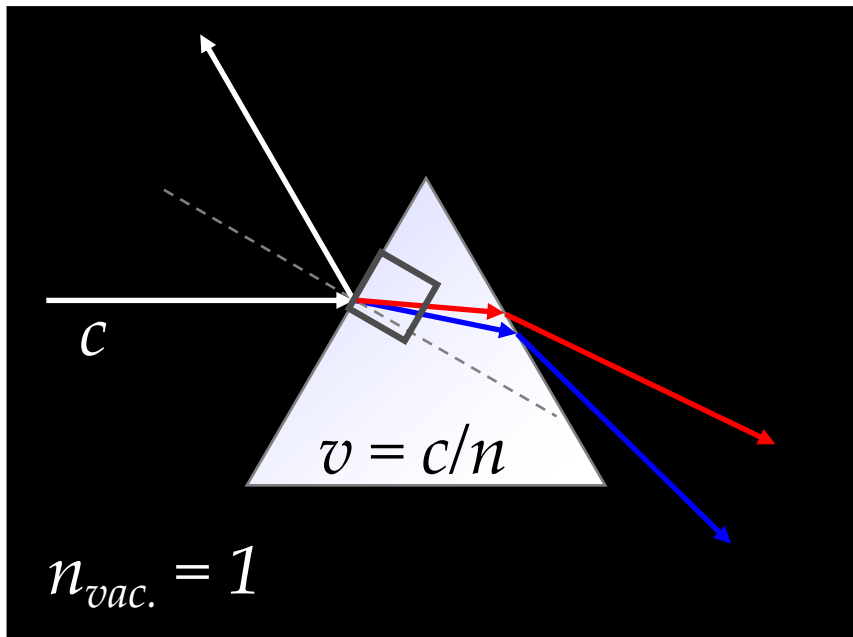


*→ : Inductive dipole moment ⇔
Receiver-transmitter **antenna**
Succession of absorption and emission*

4 – LIGHT-MATTER INTERACTION

Physical color → « *Elastic diffusion* »

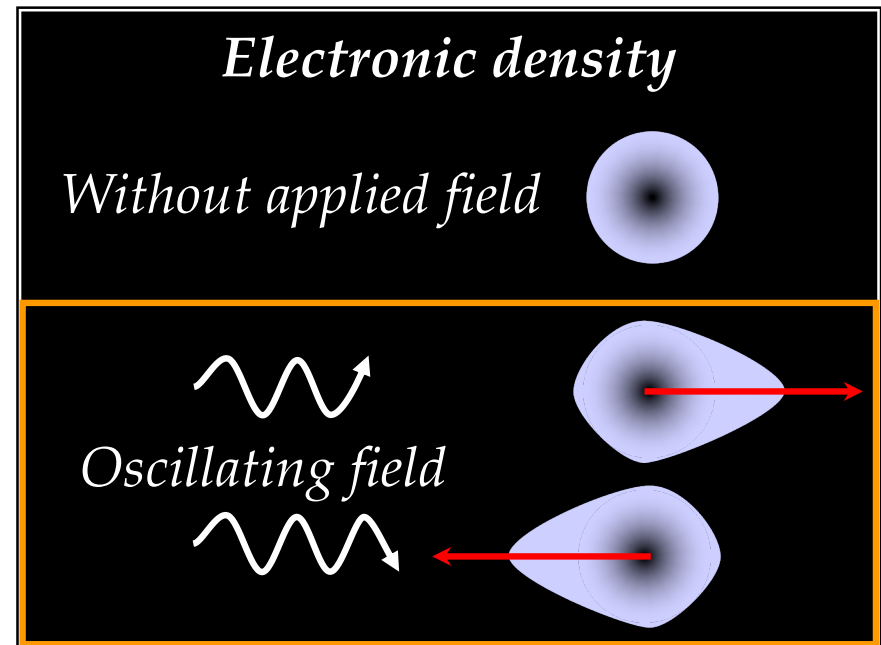
Example of a prism



Dipole moment

$$\vec{\mu} = -e \cdot \vec{r}$$

*Elastic diffusion mechanism
at the electronic level*

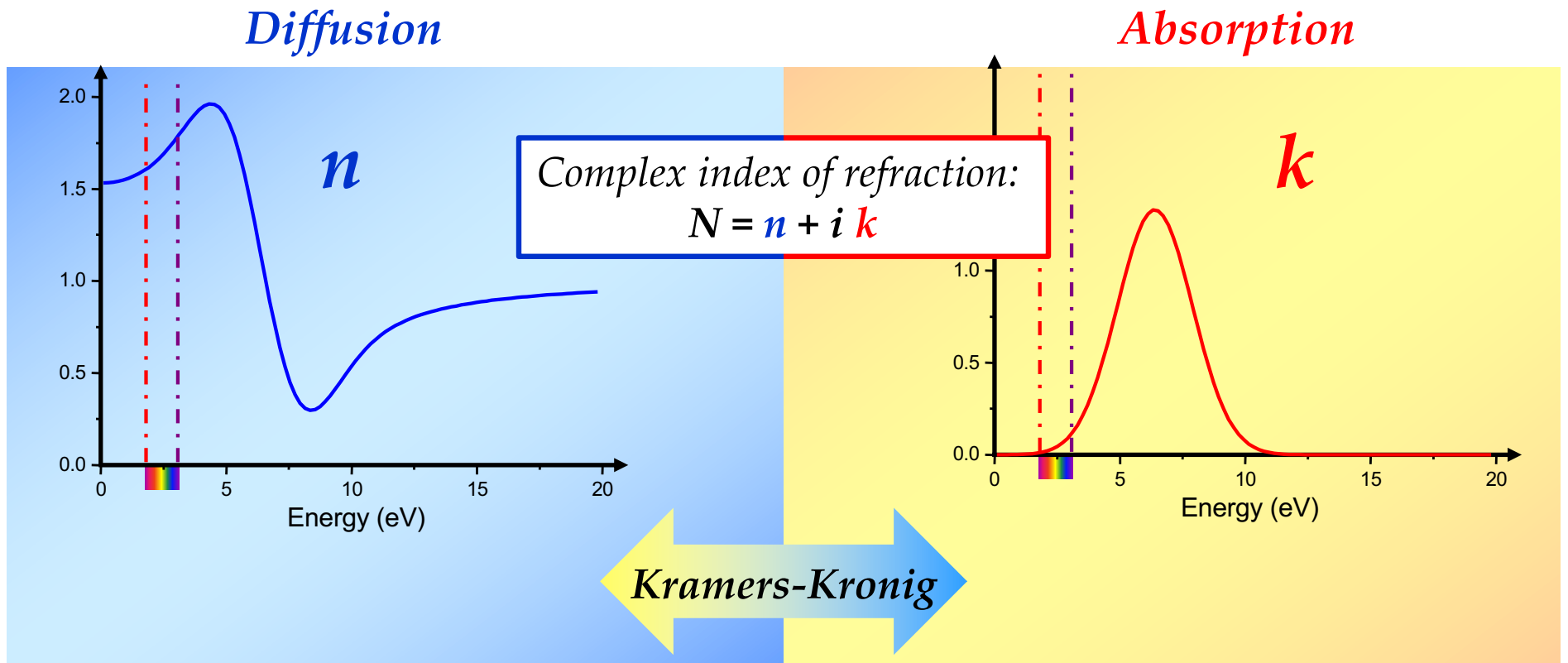


Oscillating **Electric Field** polarize ρ

→ : Inductive dipole moment \Leftrightarrow
Receiver-transmitter **antenna**

Succession of absorption and emission

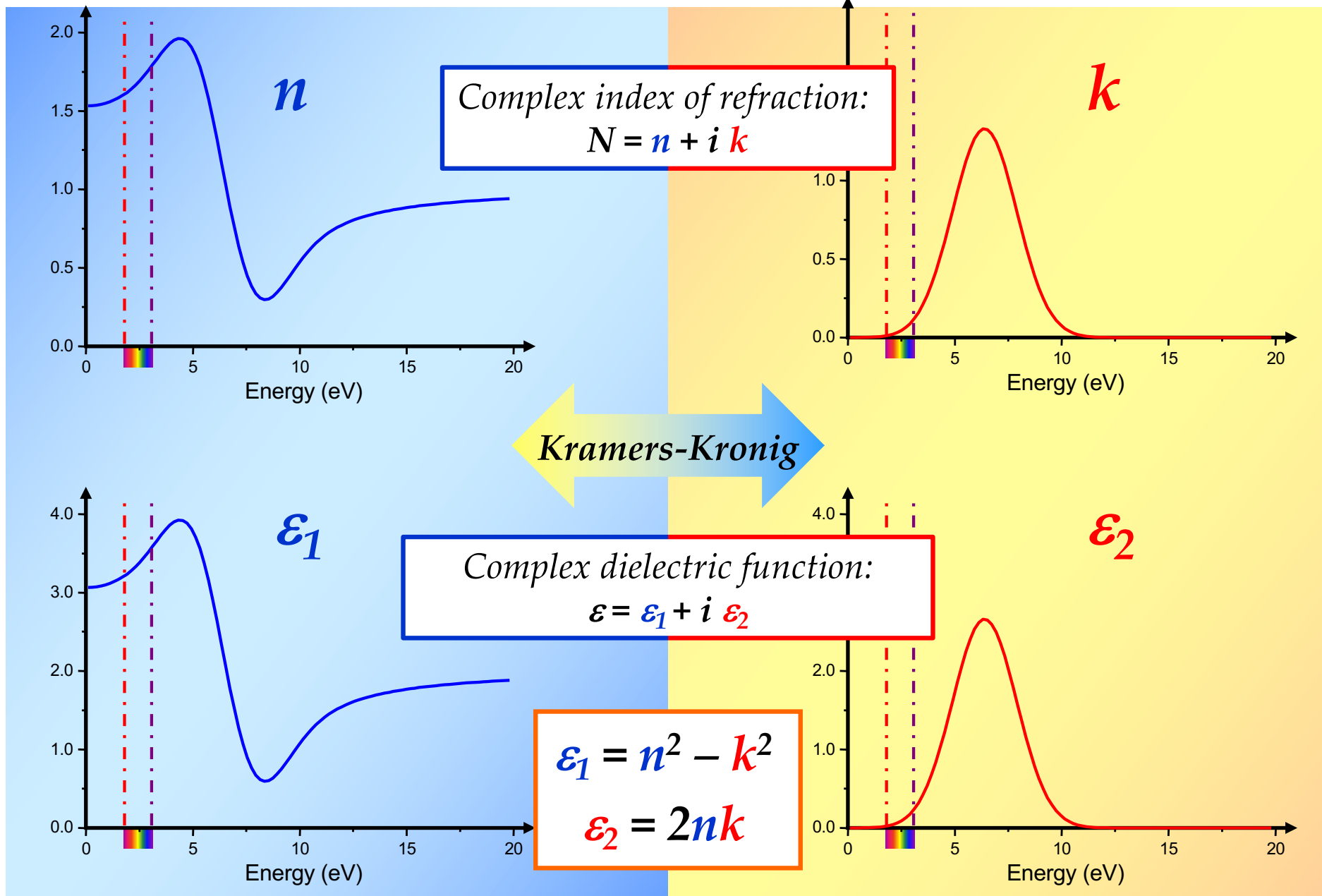
4 – LIGHT-MATTER INTERACTION



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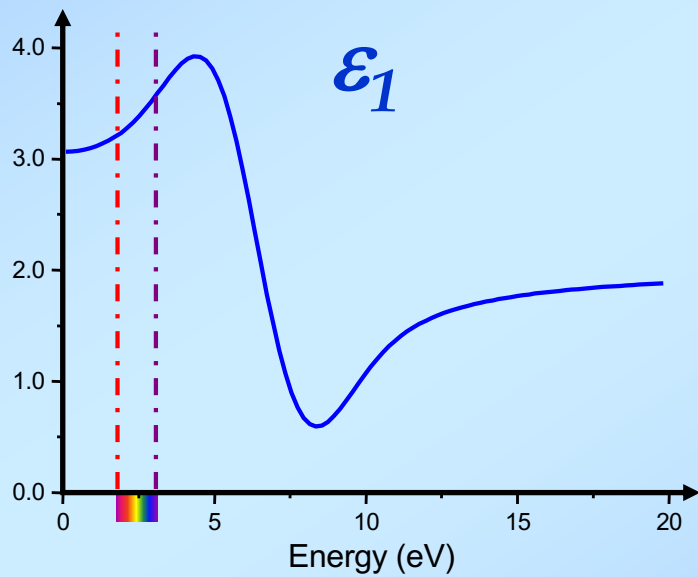
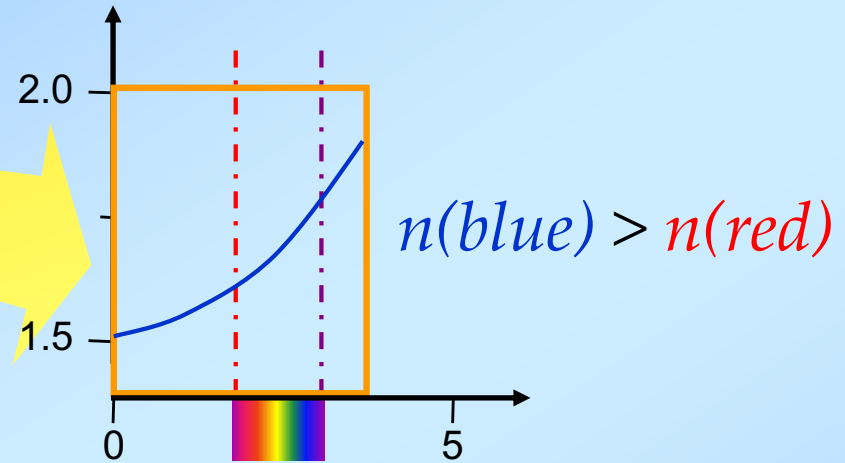
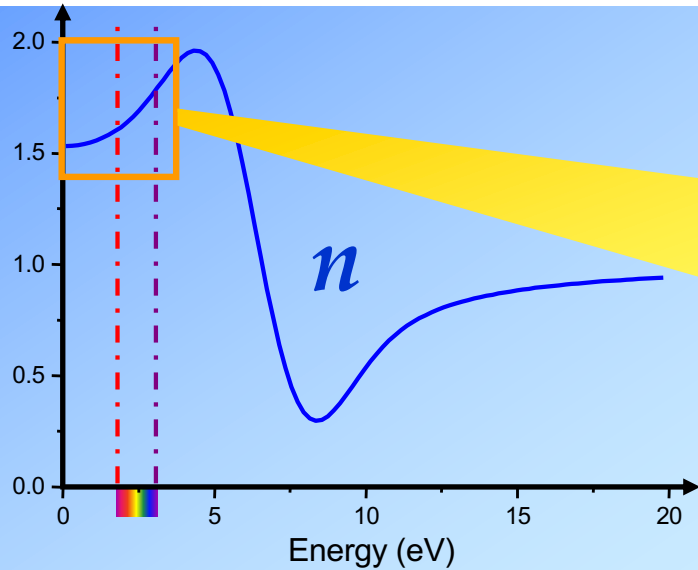
Diffusion

Absorption

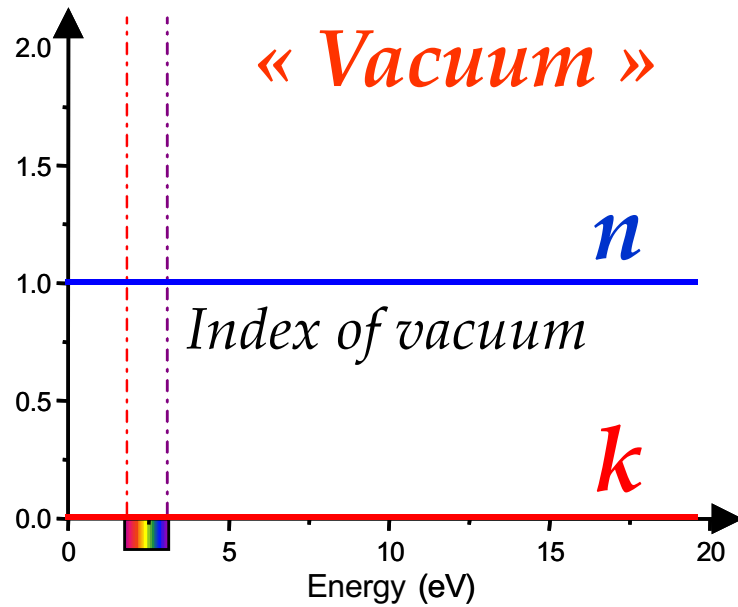


4 – LIGHT-MATTER INTERACTION

Diffusion in the case of a prism



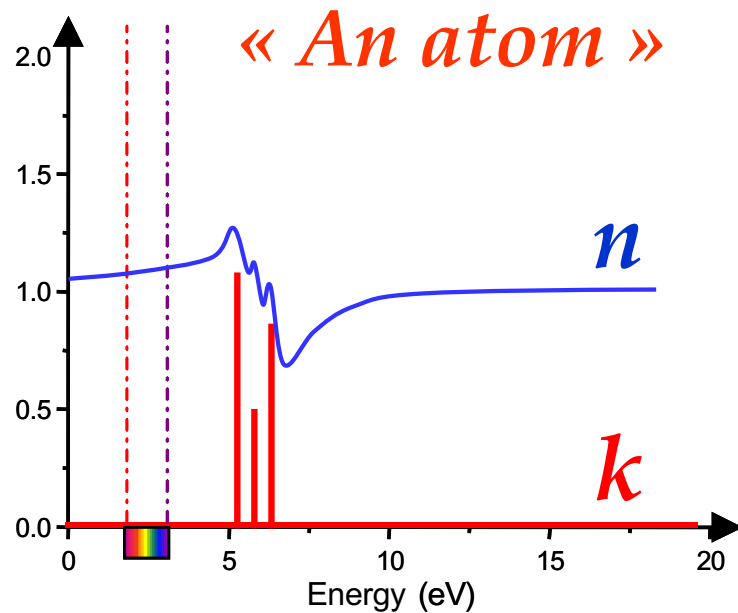
4 – LIGHT-MATTER INTERACTION



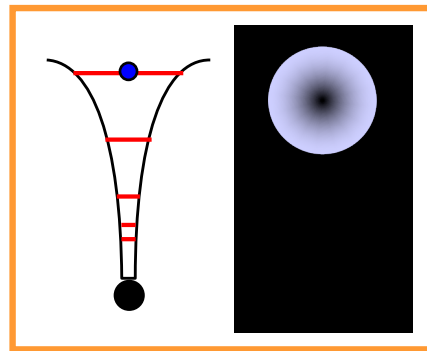
Index of refraction :
 $n = 1$

↑

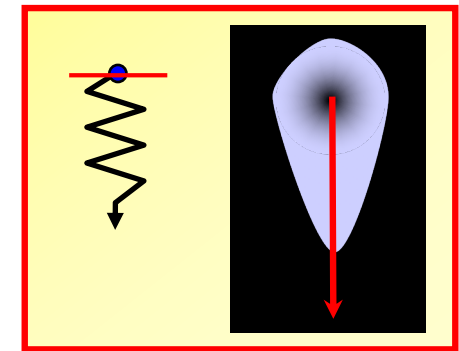
No Absorption



Without applied
electric field

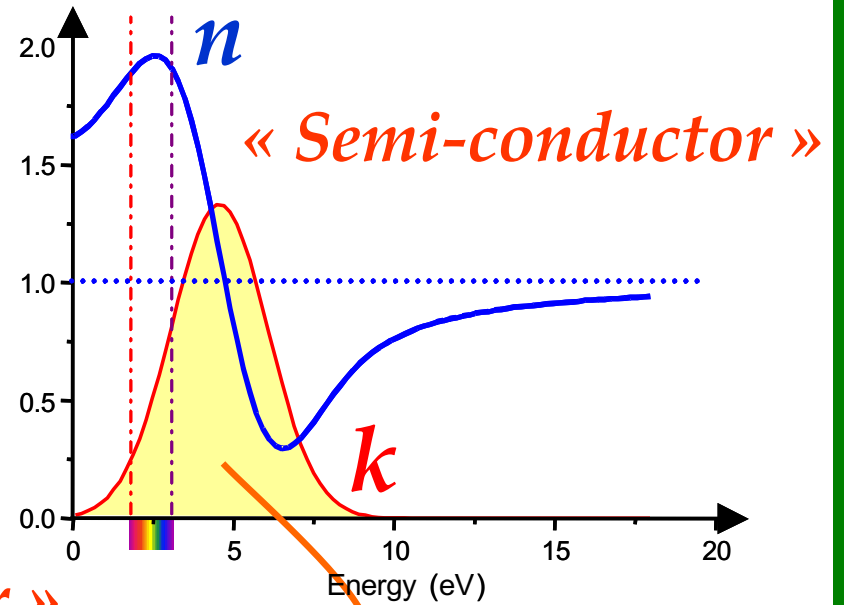
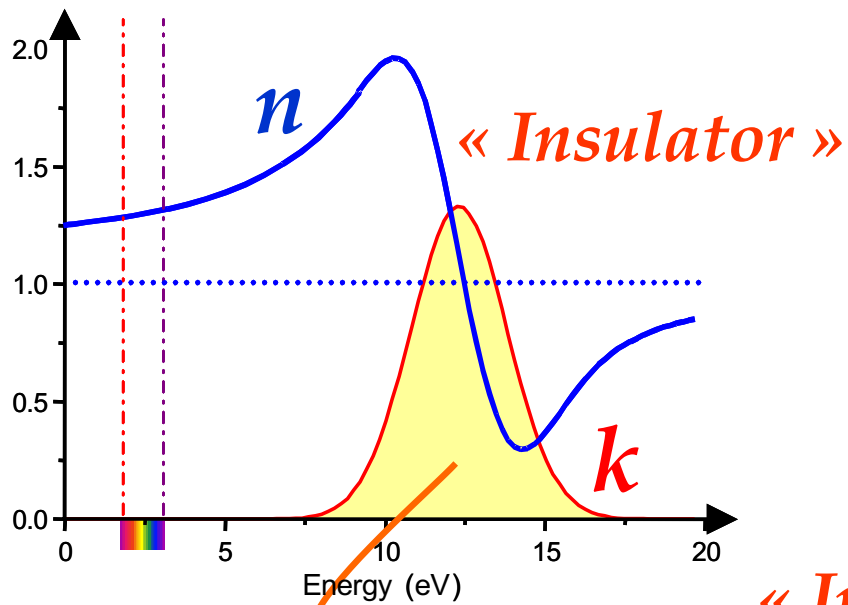


Answer to an applied
electric field



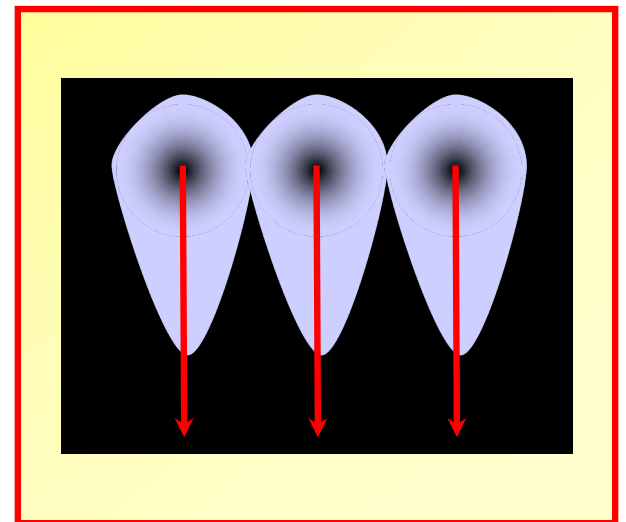
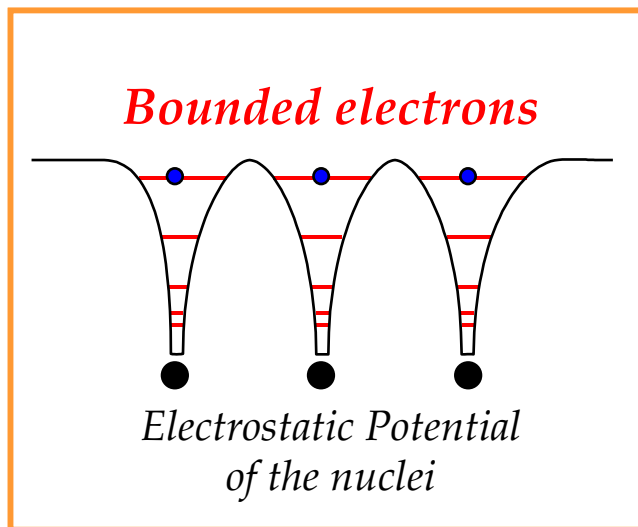
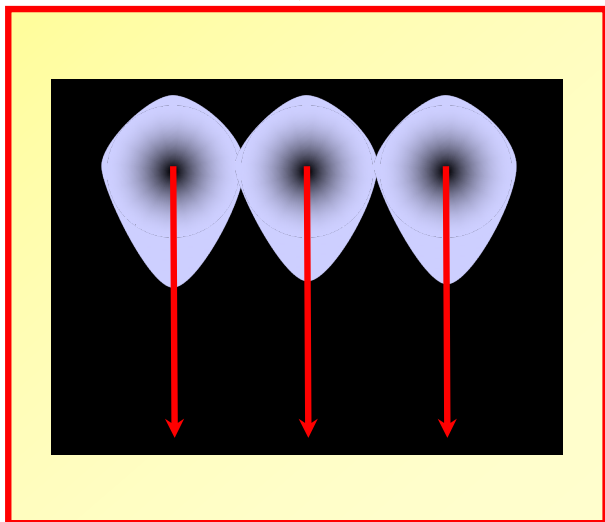
« Spring »

4 – LIGHT-MATTER INTERACTION

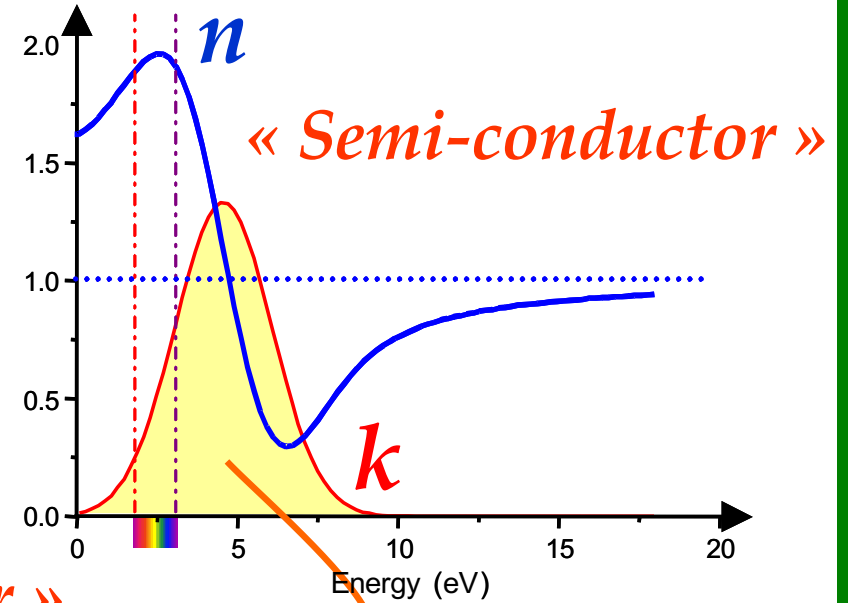
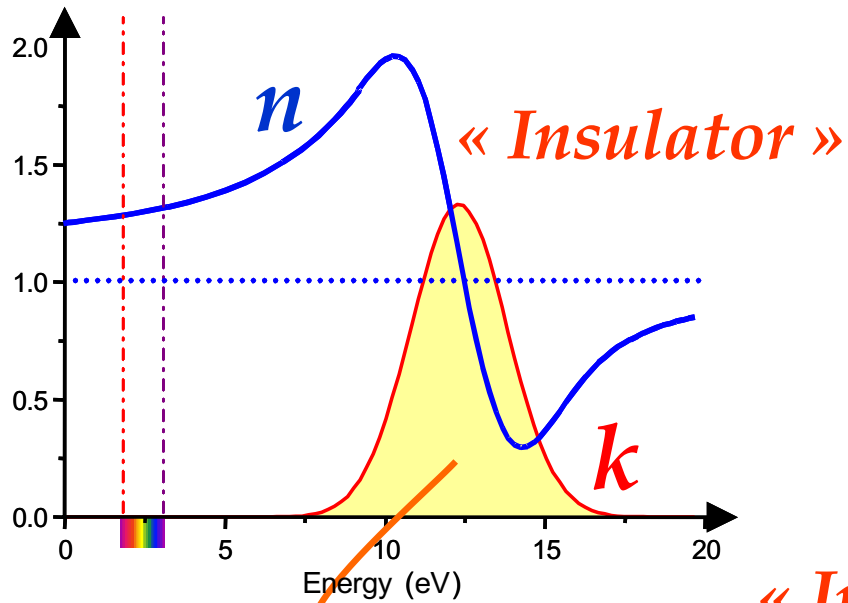


« Insulator »

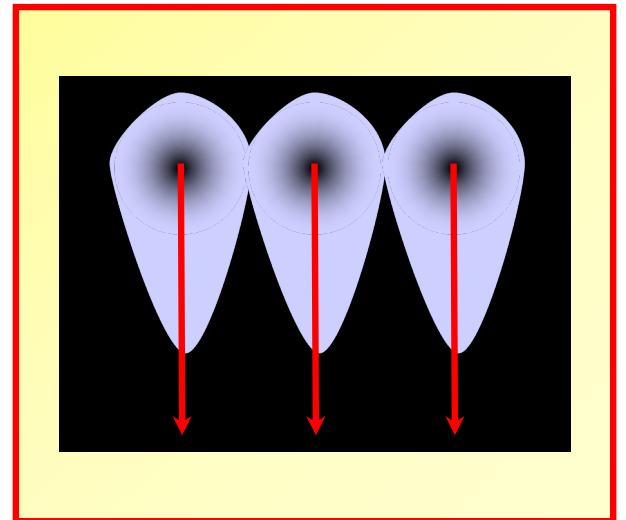
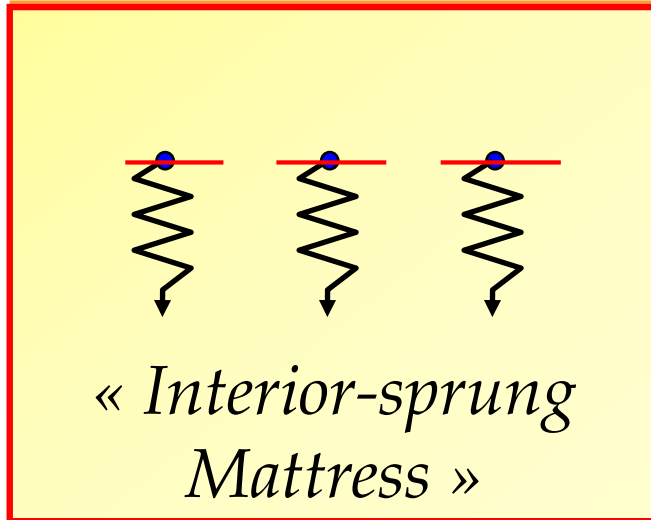
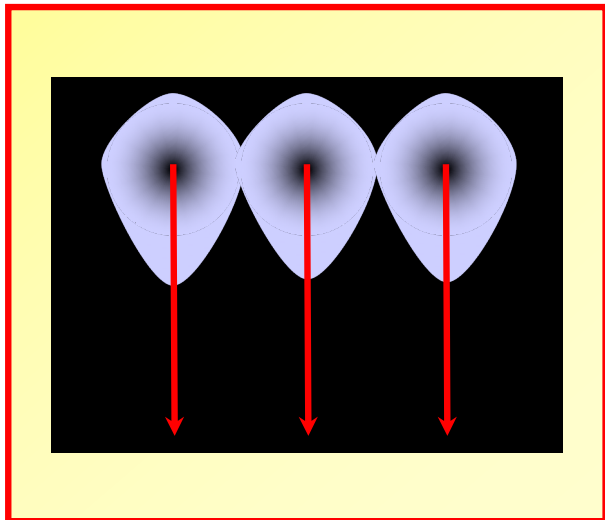
« Semi-conductor »



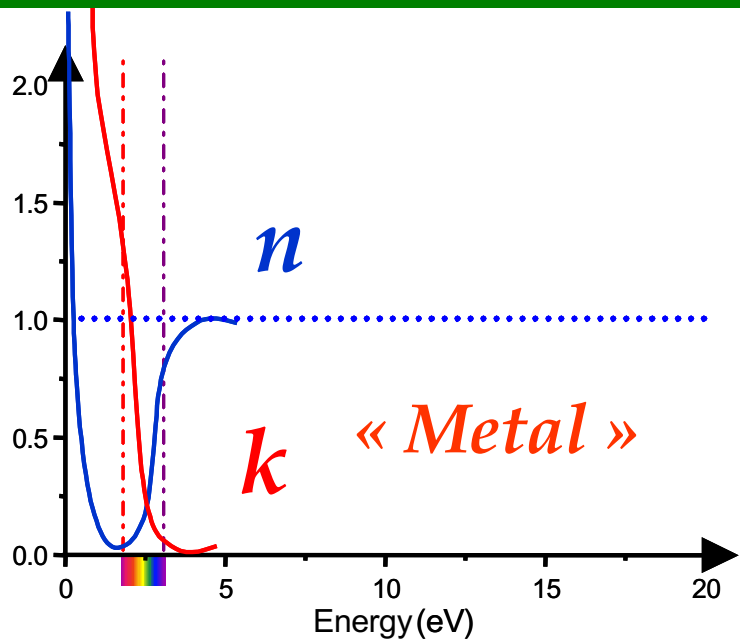
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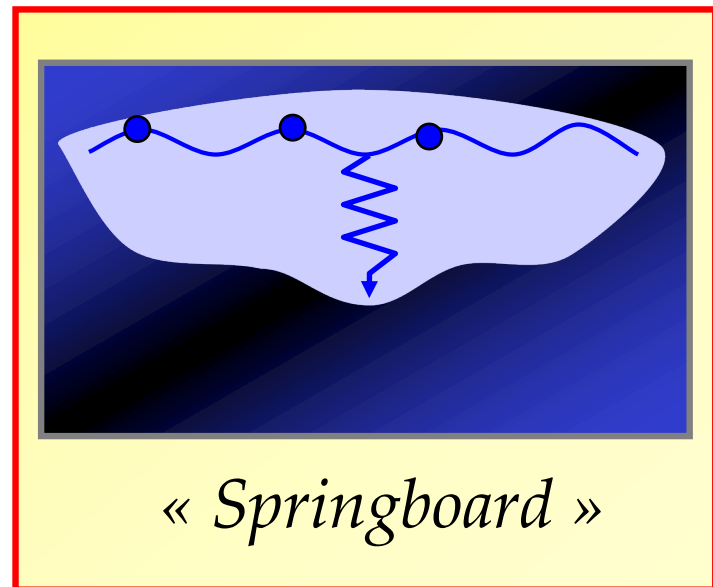
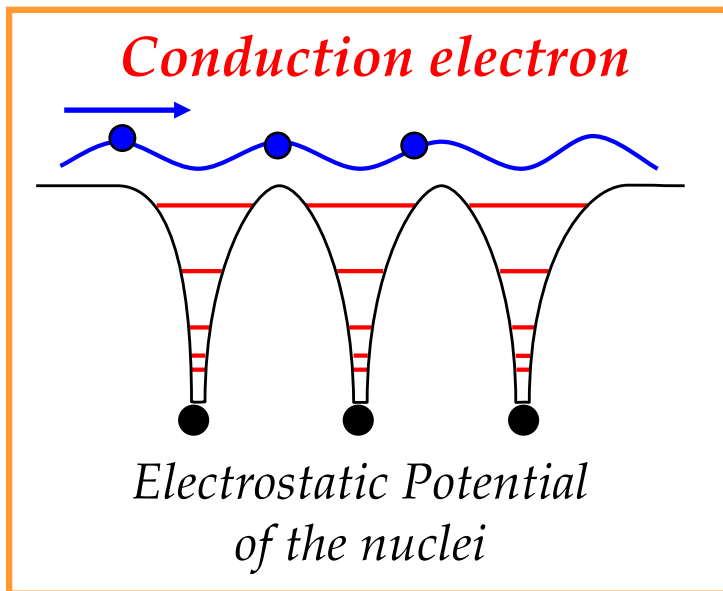
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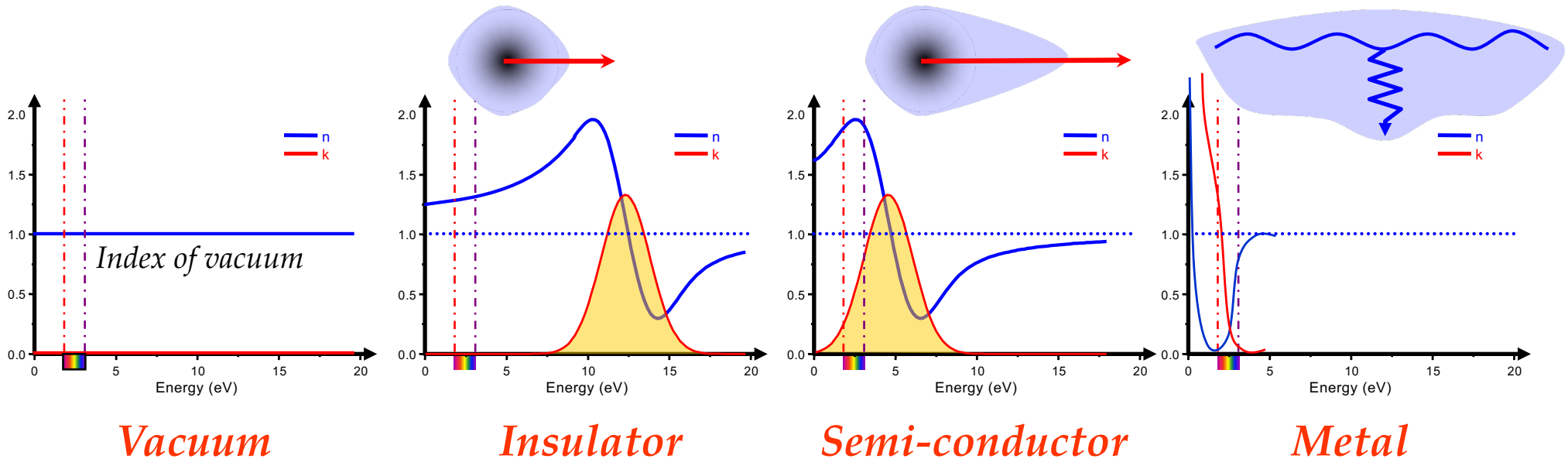
4 – LIGHT-MATTER INTERACTION



Collective answer of an optical excitation



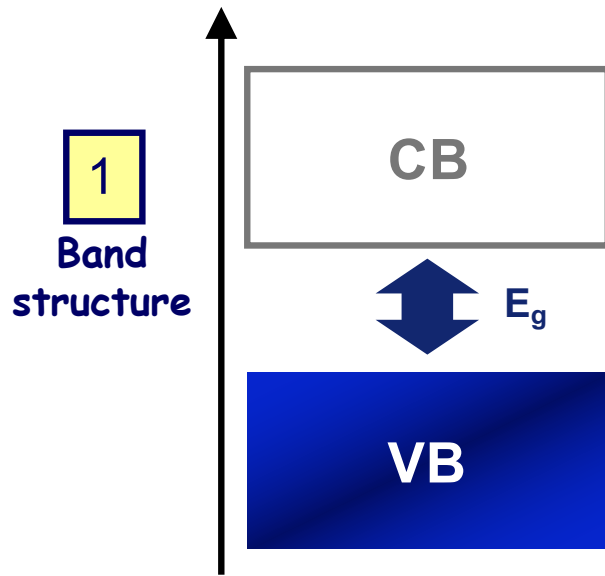
4 – LIGHT-MATTER INTERACTION



Absorption band is displaced towards low energies
Directly related to the oscillator strength

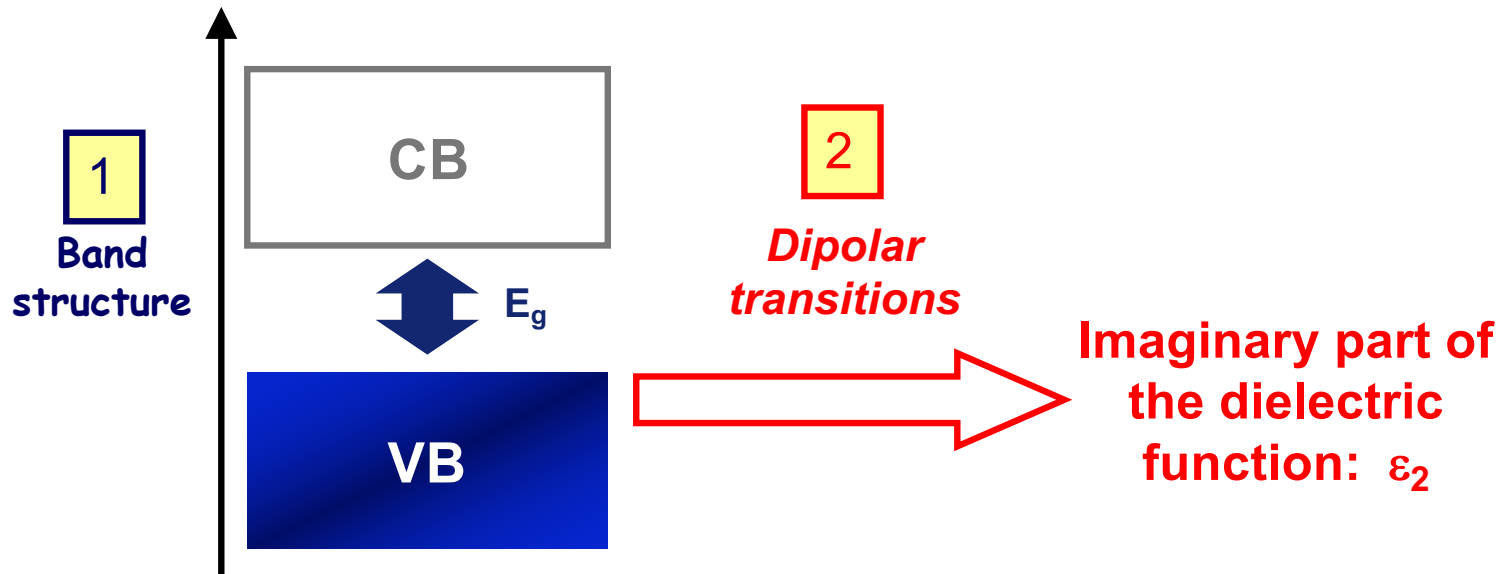
5 – OPTICAL PROPERTIES: WHICH TREATMENT?

The different steps to calculate the optical properties:



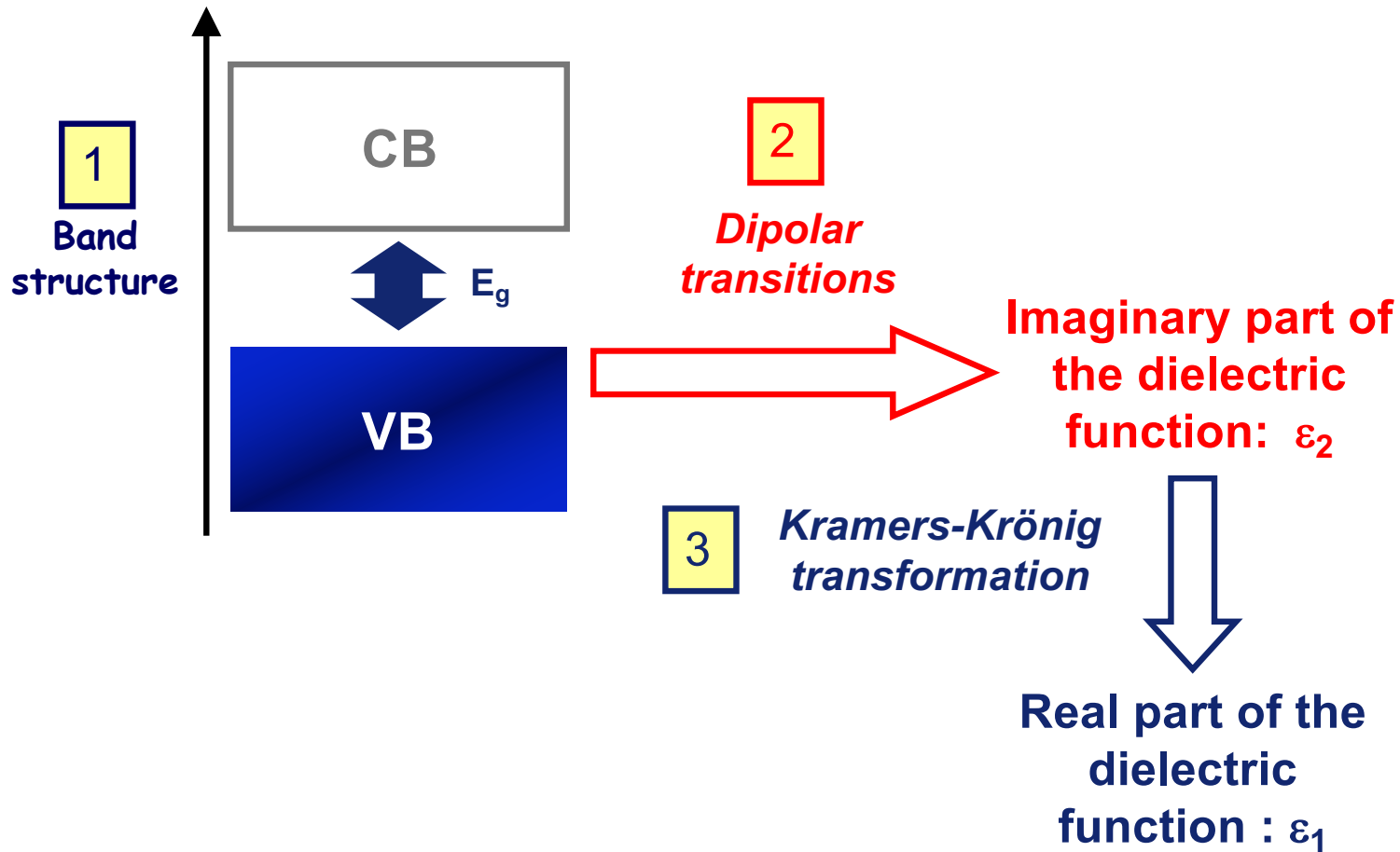
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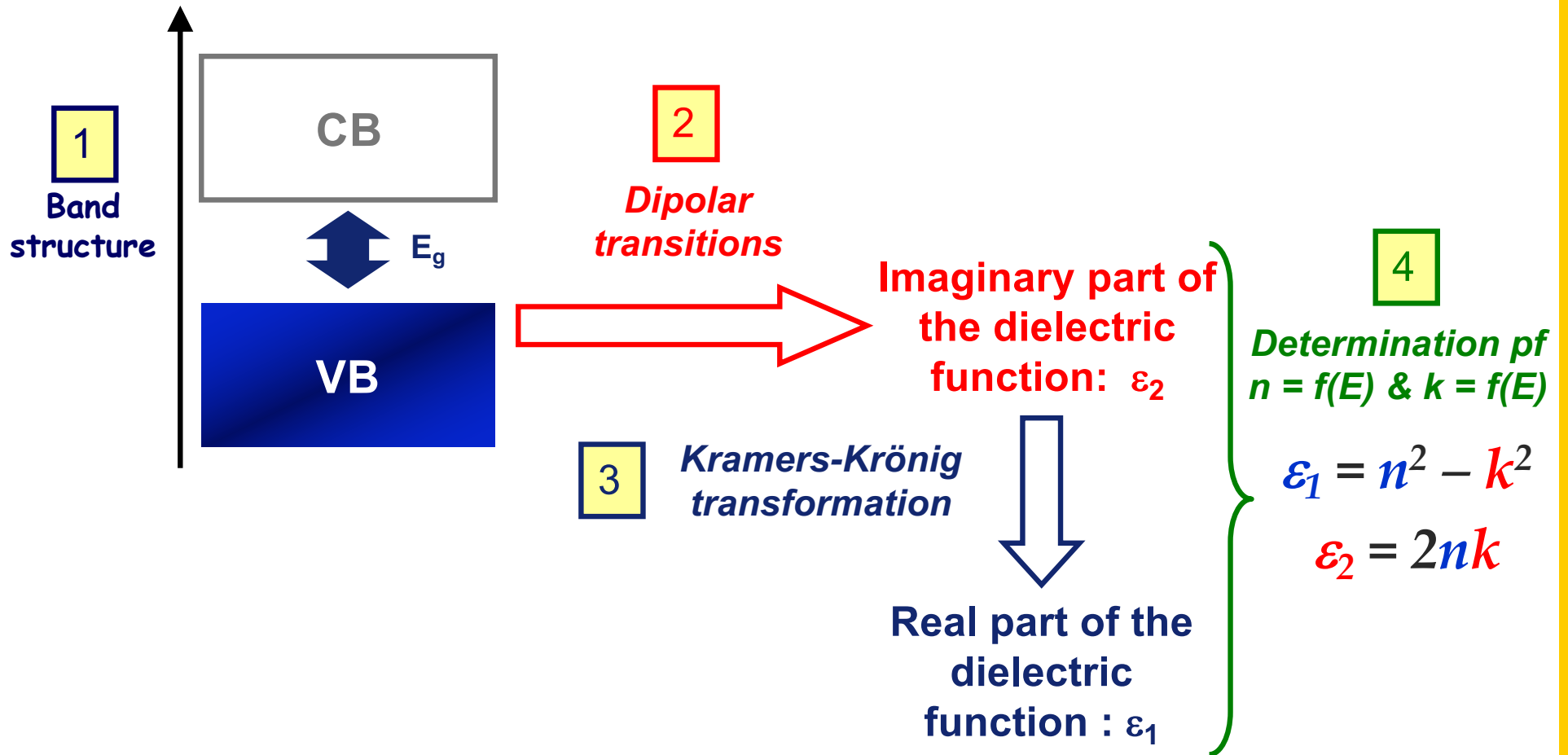
5 – OPTICAL PROPERTIES: WHICH TREATMENT?

The different steps to calculate the optical properties:



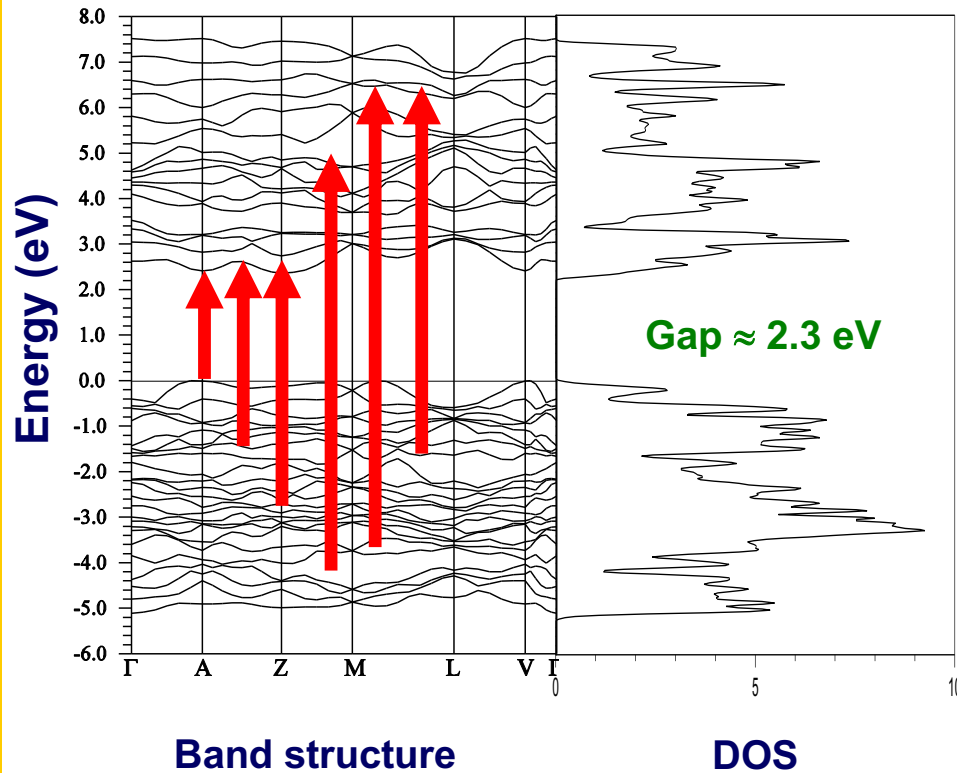
5 – OPTICAL PROPERTIES: WHICH TREATMENT?

The different steps to calculate the optical properties:

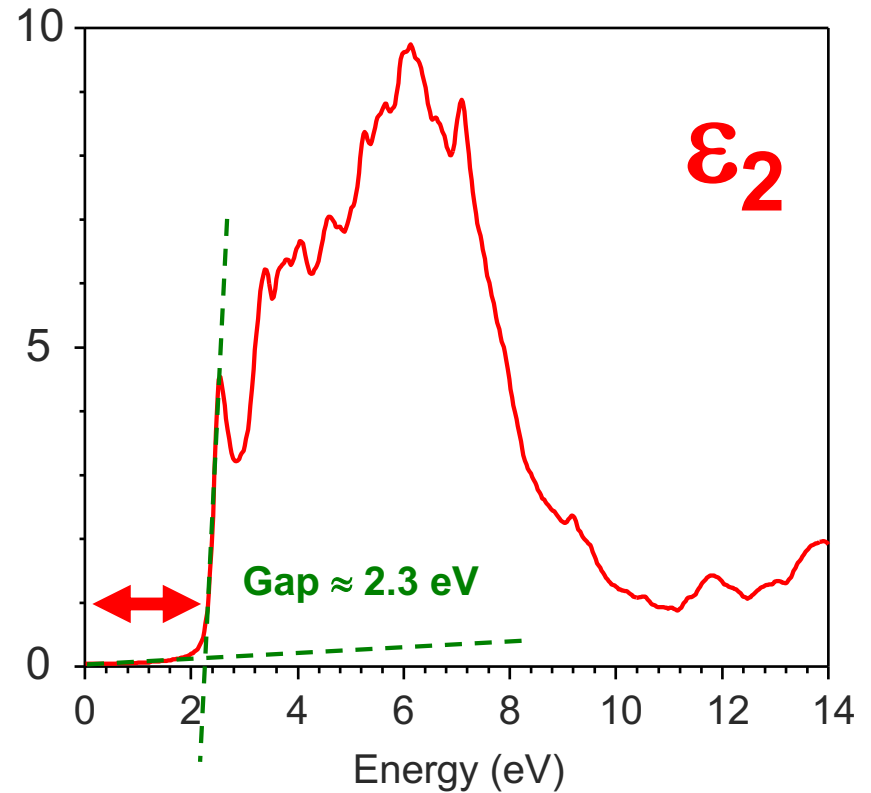


5 – OPTICAL PROPERTIES: WHICH TREATMENT?

Example of BiVO_4



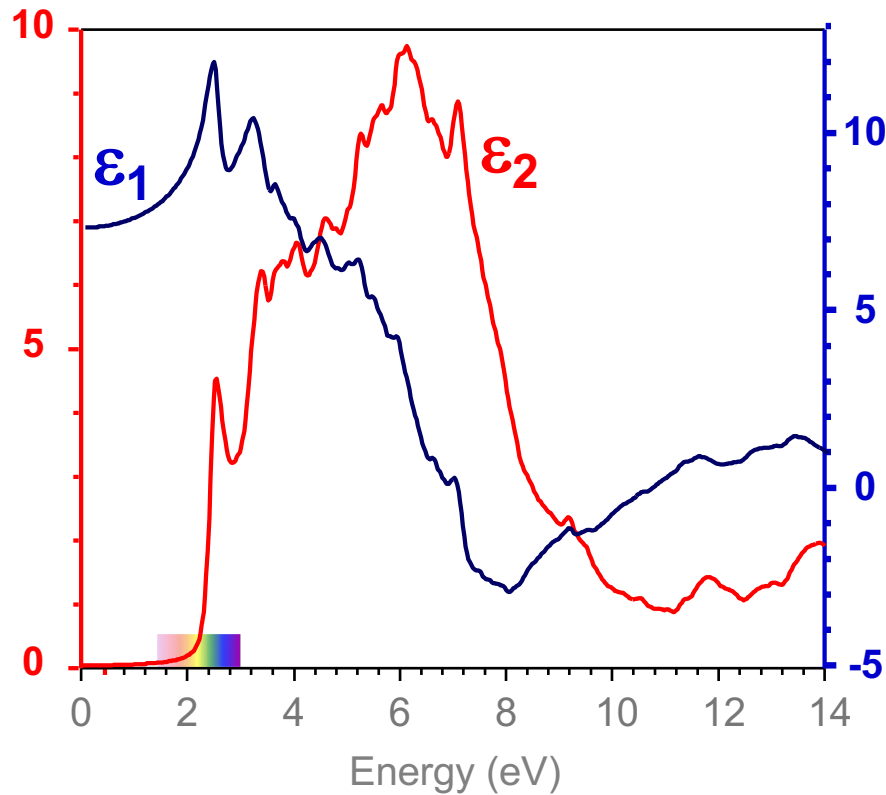
1 Band structure calculation



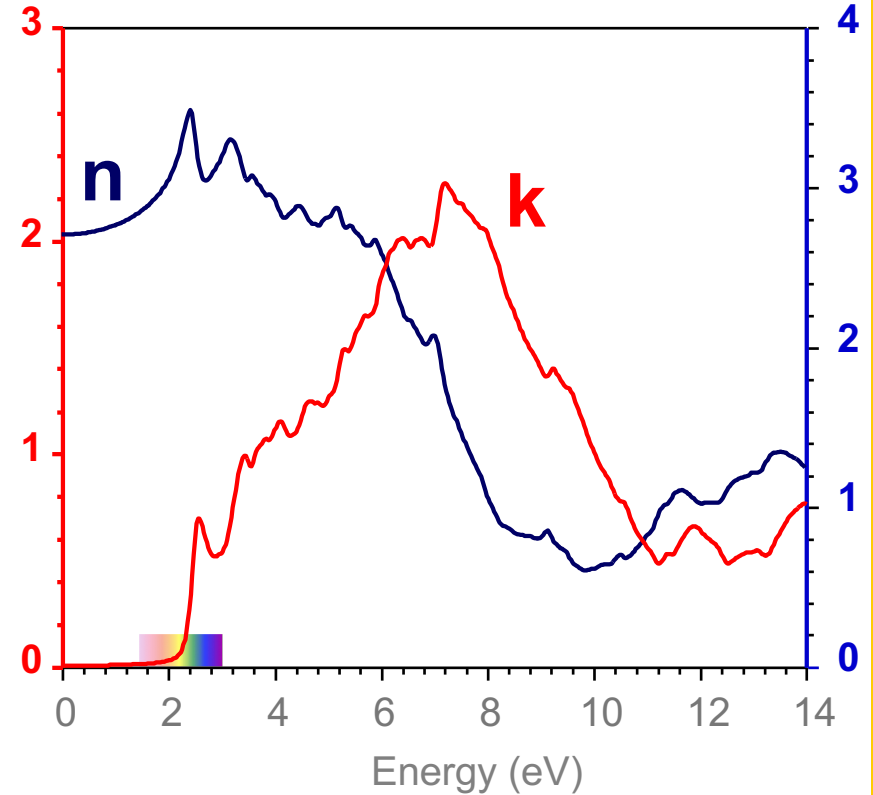
2 Determination of ϵ_2

5 – OPTICAL PROPERTIES: WHICH TREATMENT?

Example of BiVO₄



3 Determination of ϵ_1

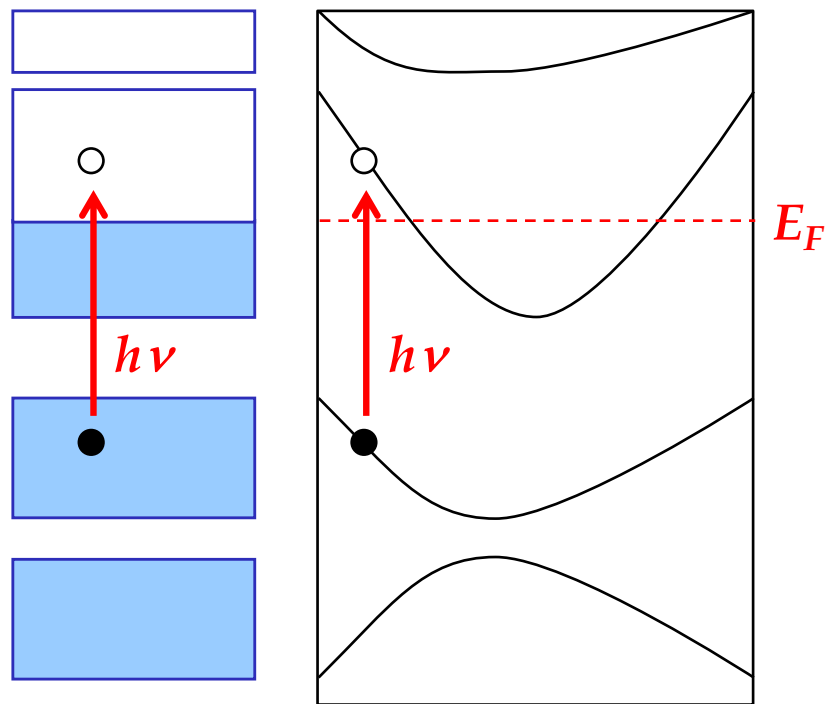


4 Determination of $n = f(E)$ & $k = f(E)$

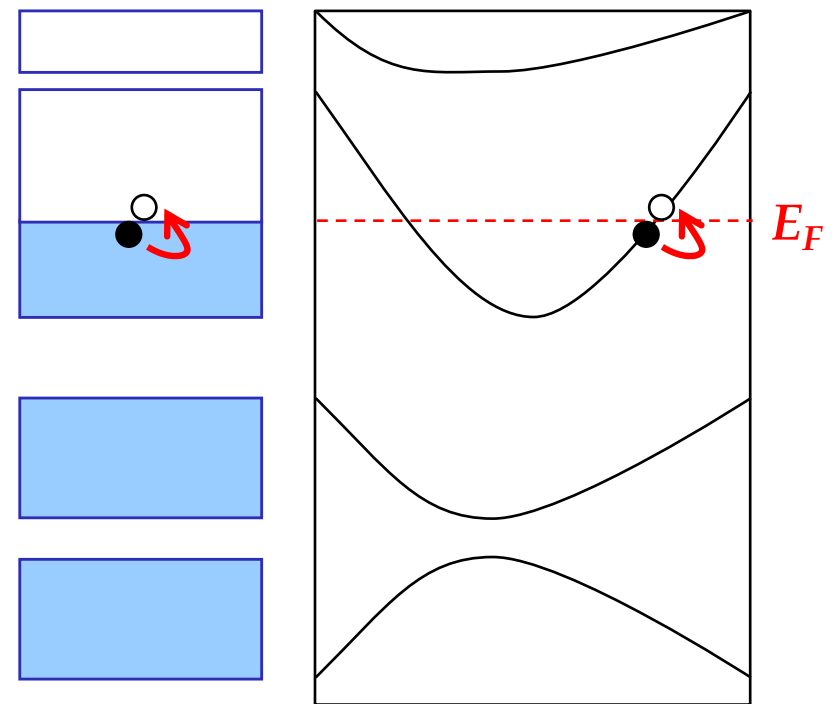
5 – OPTICAL PROPERTIES SIMULATION IN WIEN2k

In WIEN2k, two types of contributions to the dielectric function ($\epsilon = \epsilon_1 + i.\epsilon_2$) could be estimated:

*Interband contributions
(based on IPA*)*



*Intraband contributions
(using a Drude-like term)*



*→ Dielectric tensor / Optical conductivity / Refractive index /
Reflectivity / Absorption coefficient / Loss function (EELS)*

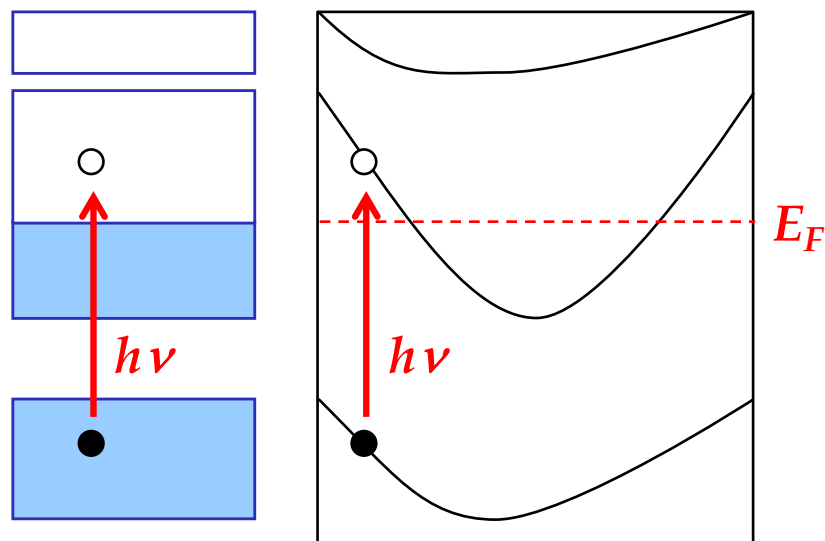
**IPA: Independant Particle Approximation*

5 – INTERPRETATION: Interband transitions

In WIEN2k, two types of contributions to the dielectric function ($\epsilon = \epsilon_1 + i.\epsilon_2$) could be estimated:

Interband contributions
(based on IPA*)

Sum over all **v**alence and **c**onduction bands



joint density of states

$$\sum_{vck} \delta(\epsilon_{kc} - \epsilon_{vk} - \omega)$$

transition probability

$$\text{Im}(\epsilon_{ij}(\omega)) = \frac{16\pi^2}{\Omega\omega^2} \sum_{vck} \langle vk | p_i | ck \rangle \langle ck | p_j | vk \rangle \delta(\epsilon_{kc} - \epsilon_{vk} - \omega)$$

*IPA: Independant Particle Approximation

5 – INTERPRETATION: Intraband transitions (for metals)

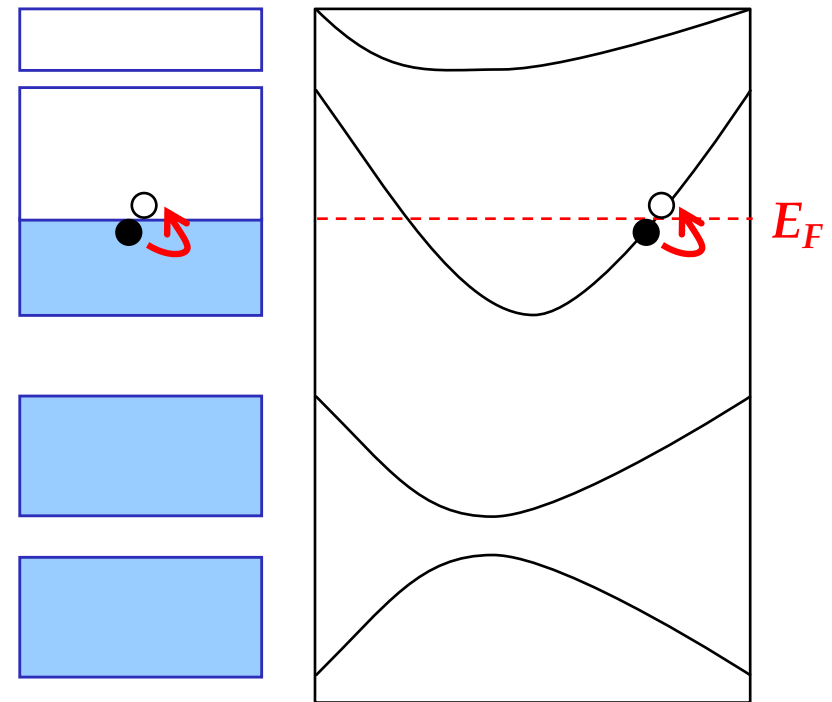
In WIEN2k, two types of contributions to the dielectric function ($\epsilon = \epsilon_1 + i.\epsilon_2$) could be estimated:

intraband: Drude model,
(ω_p : plasma frequency)

$$\text{Im } \epsilon^{\text{intra}} = \frac{\Gamma \omega_p^2}{\omega (\omega^2 + \Gamma^2)}$$

Plasma frequency: (longitudinal oscillations of the electron gas)

*Intraband contributions
(using a Drude-like term)*



$$\omega_{p,\alpha\beta}^2 = \frac{4\pi e^2}{\Omega^2} \left(\frac{n}{m} \right)_{\alpha\beta} = \frac{e^2}{m^2 \pi^2} \sum_l \int d\mathbf{k} \langle l | p^\alpha | l \rangle_{\mathbf{k}} \langle l | p^\beta | l \rangle_{\mathbf{k}} \delta(\epsilon_l - \epsilon_F)$$

5 – OPTICAL PROPERTIES SIMULATION IN WIEN2k

Optical functions:

- Dielectric tensor
$$\Im \epsilon_{ij} = \frac{16\pi^2}{\Omega \omega^2} \sum_{vck} \langle vk | p_i | ck \rangle \langle ck | p_j | vk \rangle \delta(\epsilon_{kc} - \epsilon_{vk} - \omega)$$
$$\Re \epsilon_{ij} = \delta_{ij} + \frac{2}{\pi} P \int_0^\infty \frac{\omega' \Im \epsilon_{ij}(\omega')}{\omega'^2 - \omega^2} d\omega'$$

- Optical conductivity
$$\Re \sigma_{ij}(\omega) = \frac{\omega}{4\pi} \Im \epsilon_{ij}(\omega)$$

- Refractive index
$$n_{ii} = \sqrt{|\epsilon_{ii}(\omega)| + \Re \epsilon_{ii}(\omega)}$$

$$k_{ii}(\omega) = \sqrt{\frac{|\epsilon_{ii}(\omega)| - \Re \epsilon_{ii}(\omega)}{2}}$$

- Reflectivity
$$R_{ii}(\omega) = \frac{(m_{ii} - 1)^2 + k_{ii}^2}{(m_{ii} + 1)^2 + k_{ii}^2}$$

- Absorption
$$A_{ii}(\omega) = \frac{2\omega k_{ii}(\omega)}{c}$$

- Loss function
$$L_{ii}(\omega) = -\Im \left(\frac{1}{\epsilon_{ii}(\omega)} \right)$$

5 – OPTICAL PROPERTIES SIMULATION IN WIEN2k

Symmetry of the dielectric tensor

triclinic

$$\begin{pmatrix} \text{Im } \epsilon_{xx} & \text{Im } \epsilon_{xy} & \text{Im } \epsilon_{xz} \\ \text{Im } \epsilon_{xy} & \text{Im } \epsilon_{yy} & \text{Im } \epsilon_{yz} \\ \text{Im } \epsilon_{xz} & \text{Im } \epsilon_{yz} & \text{Im } \epsilon_{zz} \end{pmatrix}$$

monoclinic ($\alpha, \beta = 90^\circ$)

$$\begin{pmatrix} \text{Im } \epsilon_{xx} & \text{Im } \epsilon_{xy} & 0 \\ \text{Im } \epsilon_{xy} & \text{Im } \epsilon_{yy} & 0 \\ 0 & 0 & \text{Im } \epsilon_{zz} \end{pmatrix}$$

orthorhombic

$$\begin{pmatrix} \text{Im } \epsilon_{xx} & 0 & 0 \\ 0 & \text{Im } \epsilon_{yy} & 0 \\ 0 & 0 & \text{Im } \epsilon_{zz} \end{pmatrix}$$

tetragonal, hexagonal

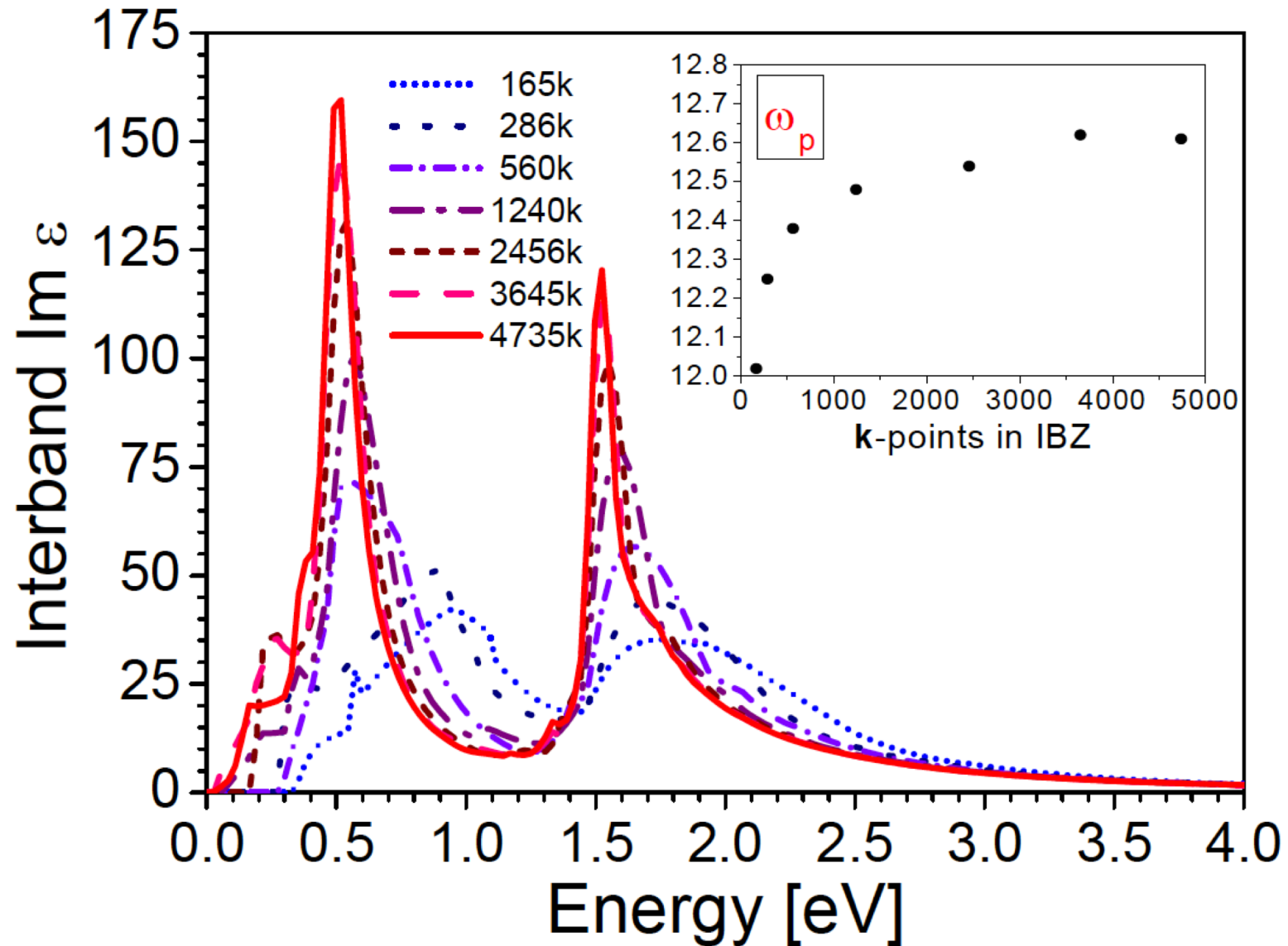
$$\begin{pmatrix} \text{Im } \epsilon_{xx} & 0 & 0 \\ 0 & \text{Im } \epsilon_{xx} & 0 \\ 0 & 0 & \text{Im } \epsilon_{zz} \end{pmatrix}$$

cubic

$$\begin{pmatrix} \text{Im } \epsilon_{xx} & 0 & 0 \\ 0 & \text{Im } \epsilon_{xx} & 0 \\ 0 & 0 & \text{Im } \epsilon_{xx} \end{pmatrix}$$

5 – OPTICAL PROPERTIES SIMULATION IN WIEN2k

Convergence with k-mesh (expl.: Al)



5 – OPTICS IN WIEN2k

- 0 normal SCF run \longrightarrow converged density
- 1 x `kgen` \longrightarrow dense `k-mesh` (check `convergence!`)
- 2 x `lapw1 -options` \longrightarrow eigenvectors on dense mesh
- 3 x `lapw2 -fermi -options` \longrightarrow `case.weight`
 - metals: “TETRA 101.0” in `case.in2`
- 4 x `optic -options` \longrightarrow momentum matrix elements
`case.symmat`: $\langle ck|\hat{p}_i|vk\rangle \langle vk|\hat{p}_j|ck\rangle$
- 5 x `joint` \rightarrow $\text{Im } \varepsilon_{ij}(\omega)$ (`case.joint`)
- 6 x `kram` \rightarrow $\text{Re } \varepsilon_{ij}(\omega)$, other optical funct.
- 7 `opticplot`

5 – SOME ADDITIONAL DETAILS

spin-polarized calculations

- 1 x joint -up && x joint -dn
- 2 addjoint-updn
- 3 x kram

procedure for metals

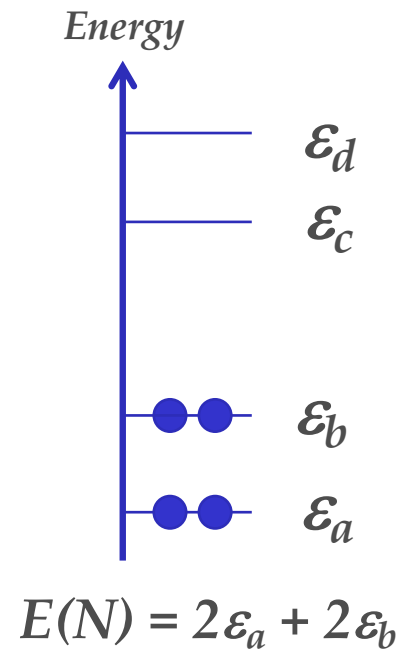
- 1 x joint (mode=6) \longrightarrow plasma frequencies $\omega_{p_{ij}}$
- 2 x joint (mode=4) \longrightarrow interband $\text{Im } \epsilon$
- 3 x kram (intra=1, insert ω_p)

Kramers-Kronig needs $\text{Im } \epsilon$ in a large energy range

$$\text{Re } \epsilon_{ij} = \delta_{ij} + \frac{2}{\pi} \mathcal{P} \int_0^{\infty} d\Omega \frac{\Omega}{\Omega^2 - \omega^2} \text{Im } \epsilon_{ij}$$

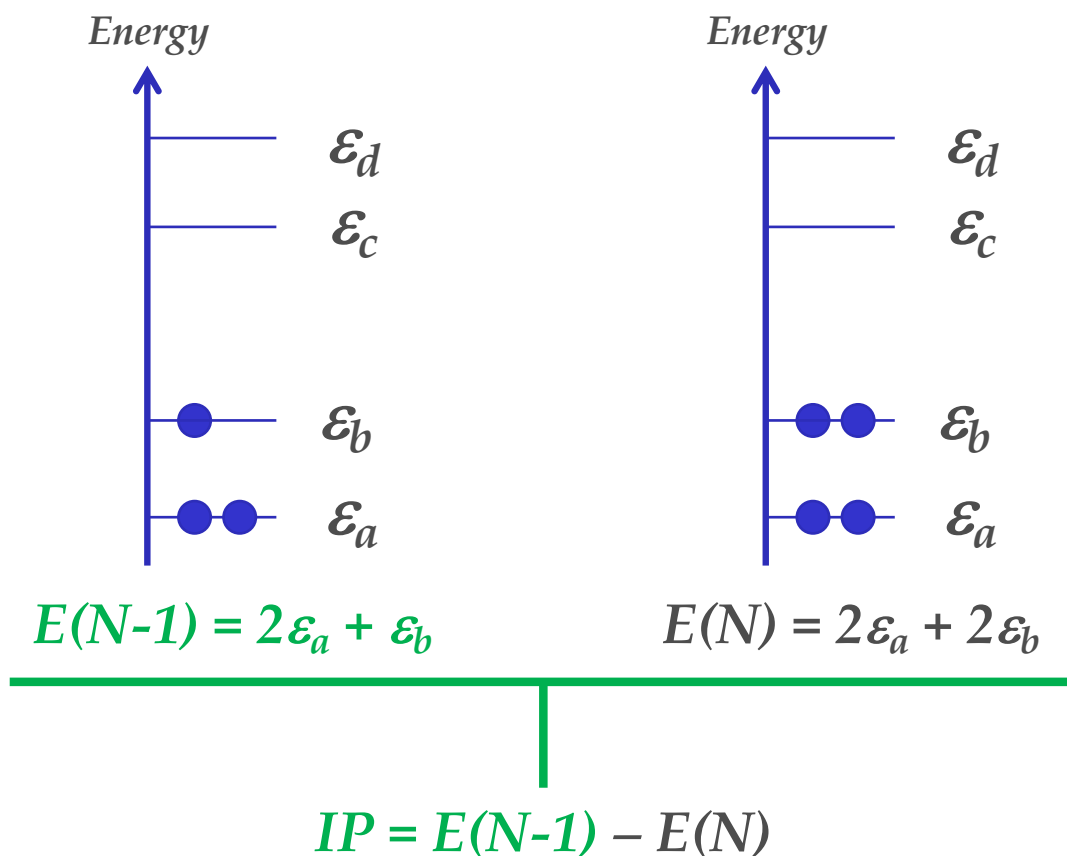
5 – INTERPRETATION: Band gap problem

The band gap problem → Necessity to go beyond DFT



5 – INTERPRETATION: Band gap problem

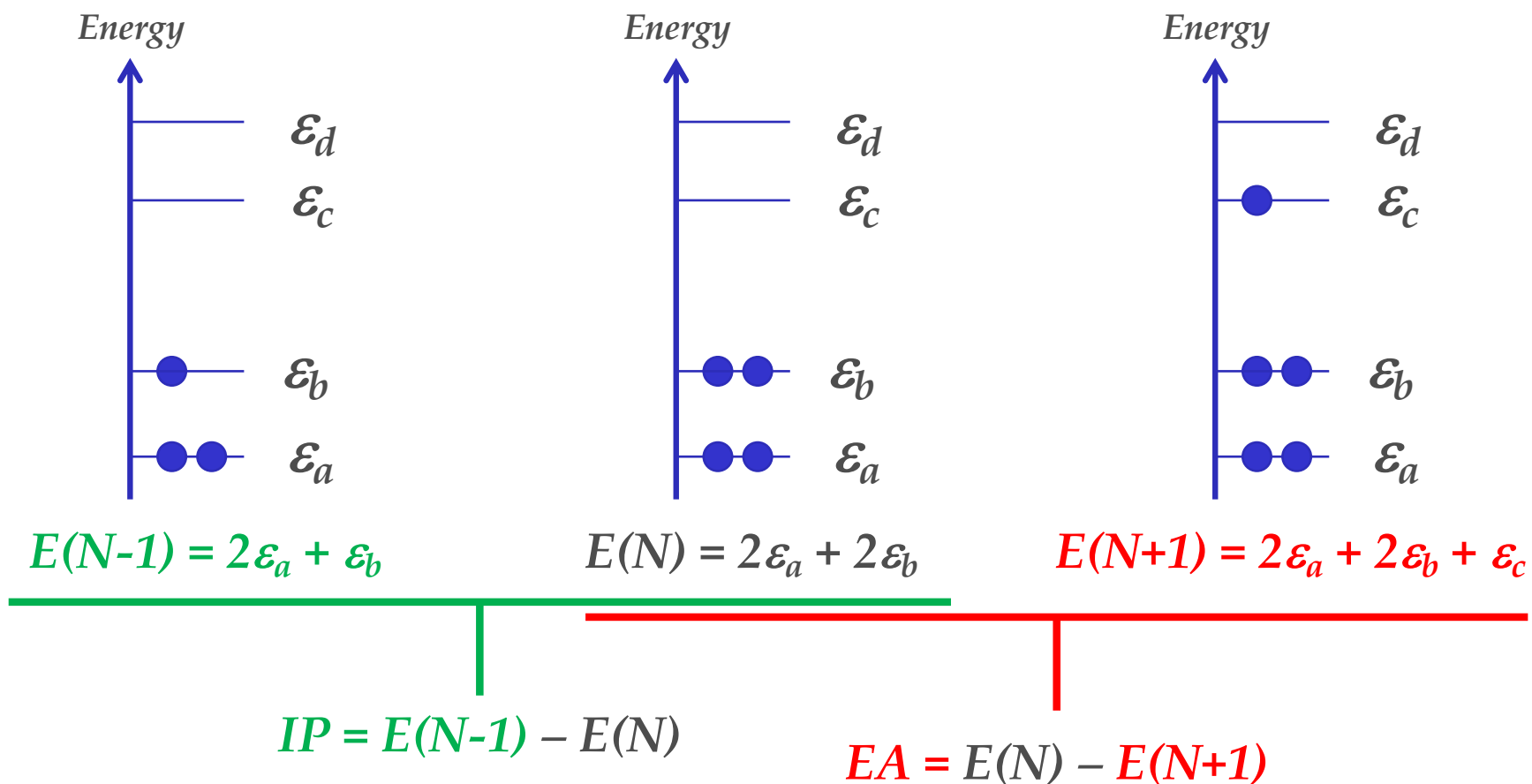
The band gap problem → Necessity to go beyond DFT



IP: Ionization potential

5 – INTERPRETATION: Band gap problem

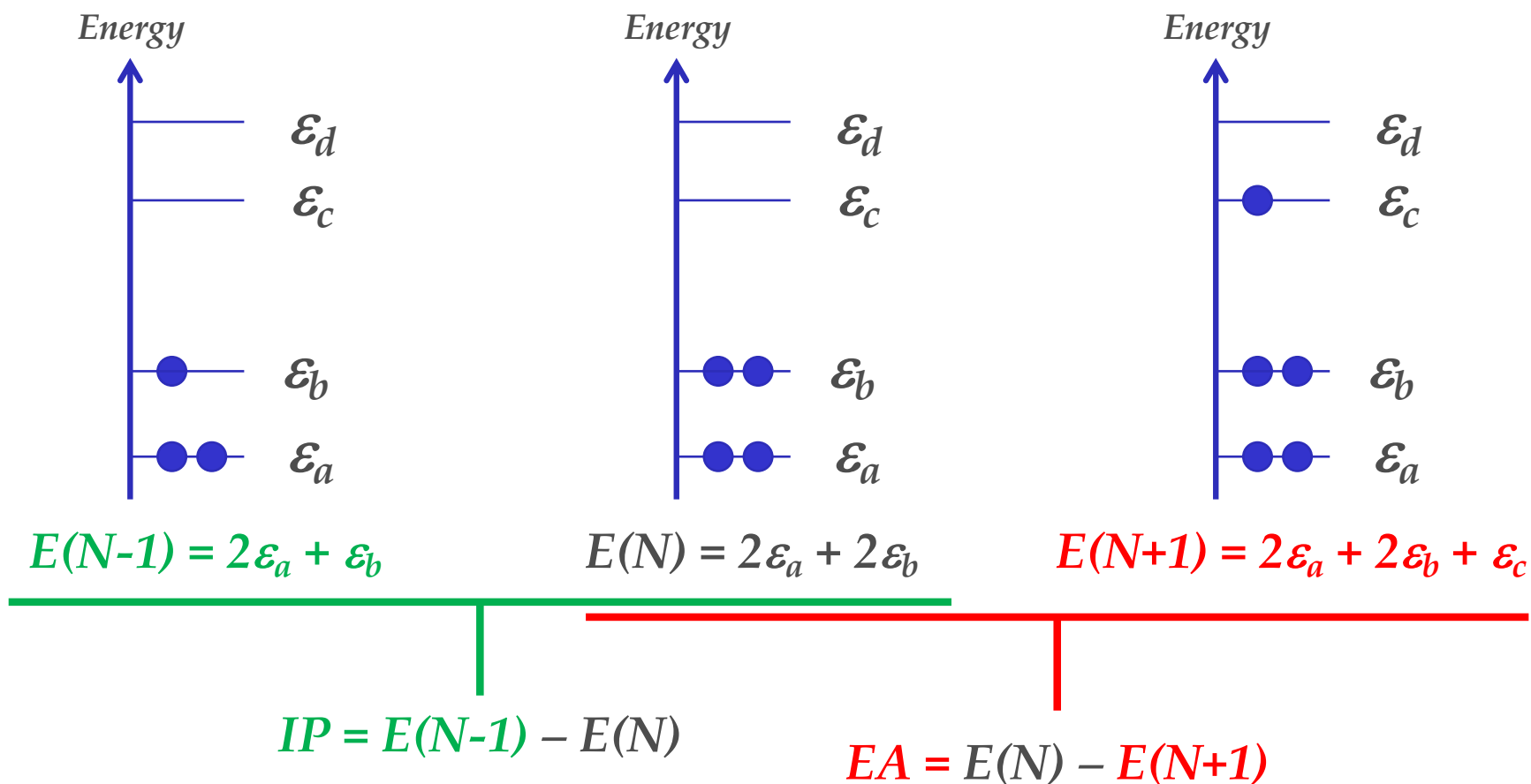
The band gap problem → Necessity to go beyond DFT



IP: Ionization potential EA: Electron affinity

5 – INTERPRETATION: Band gap problem

The band gap problem → Necessity to go beyond DFT



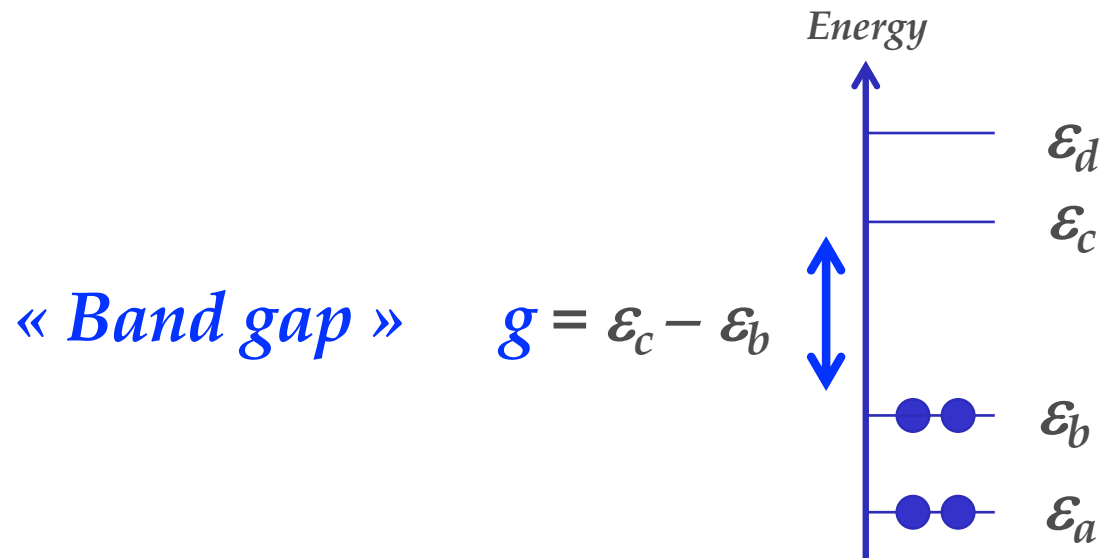
Fundamental
energy gap

$$G = IP - EA$$

$$= E(N-1) + E(N+1) - 2E(N)$$

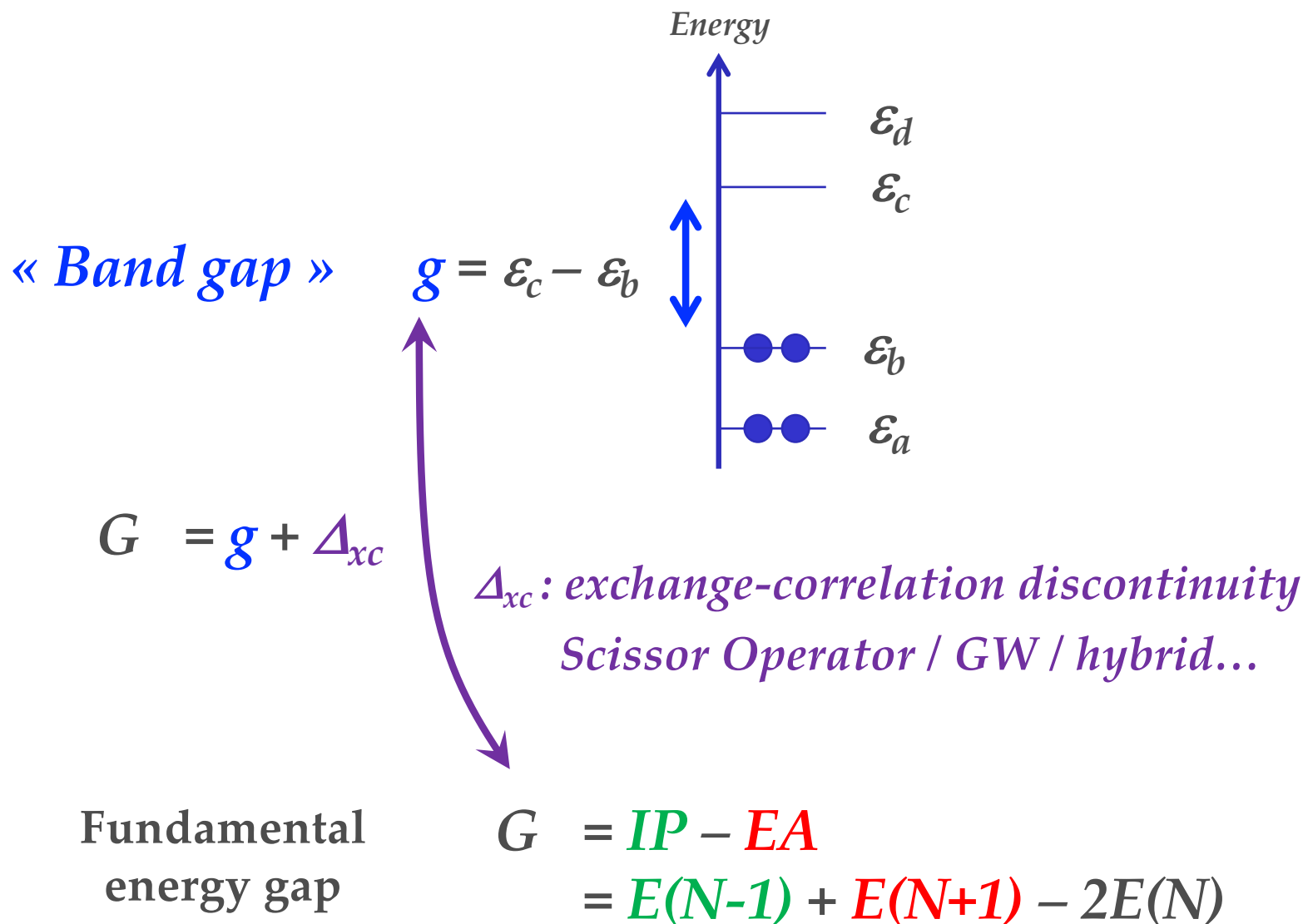
5 – INTERPRETATION: Band gap problem

The band gap problem → Necessity to go beyond DFT



5 – INTERPRETATION: Band gap problem

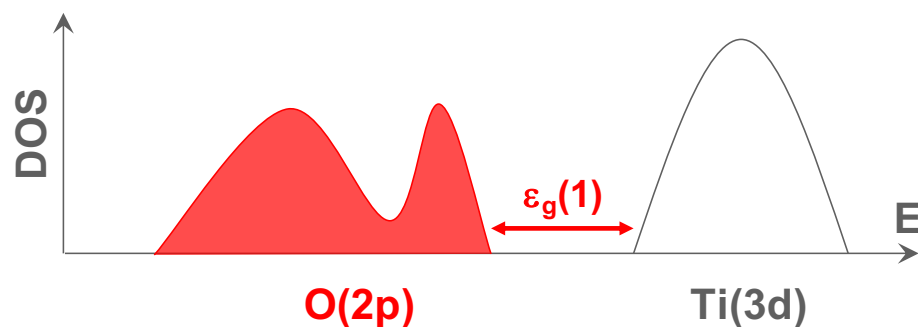
The band gap problem → Necessity to go beyond DFT



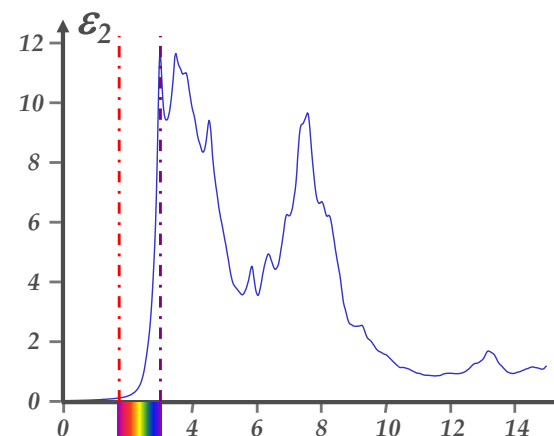
5 – OPTICAL PROPERTIES SIMULATION IN WIEN2k

Fundamental gap: electronic gap \neq optical gap

Deduced from the band structure



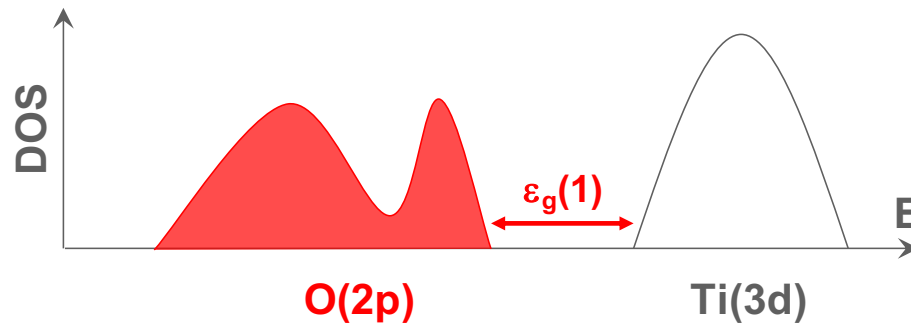
Dipolar transitions



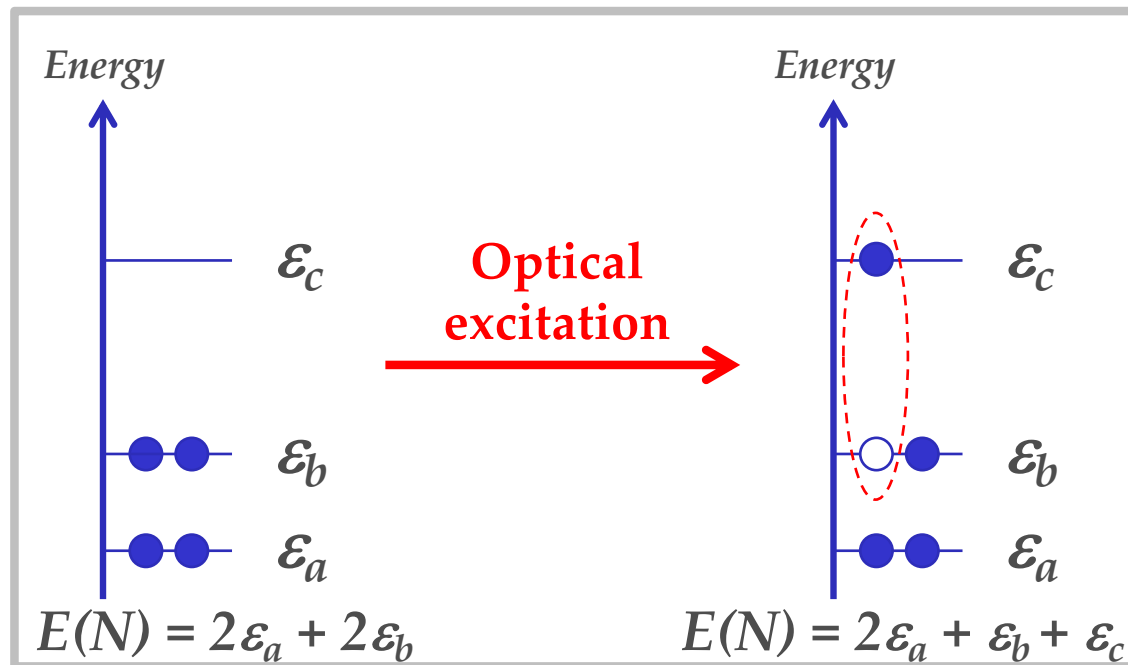
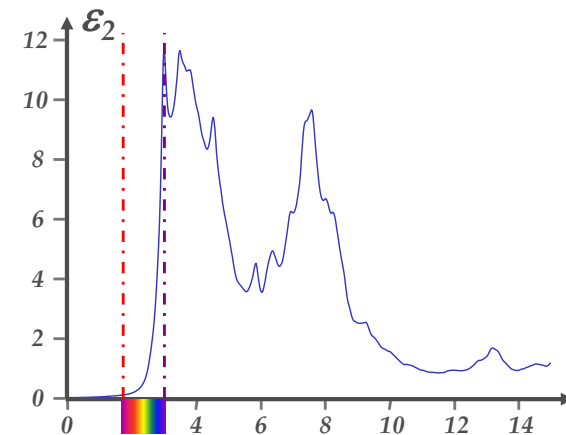
5 – OPTICAL PROPERTIES SIMULATION IN WIEN2k

Fundamental gap: electronic gap \neq optical gap

Deduced from the band structure

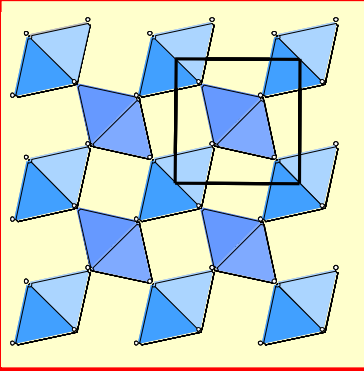


Dipolar transitions



*If excitonic effects:
we should go beyond
(TDDFT / BSE)*

6 – ILLUSTRATIONS: TiO_2 series



Example of the rutile phase

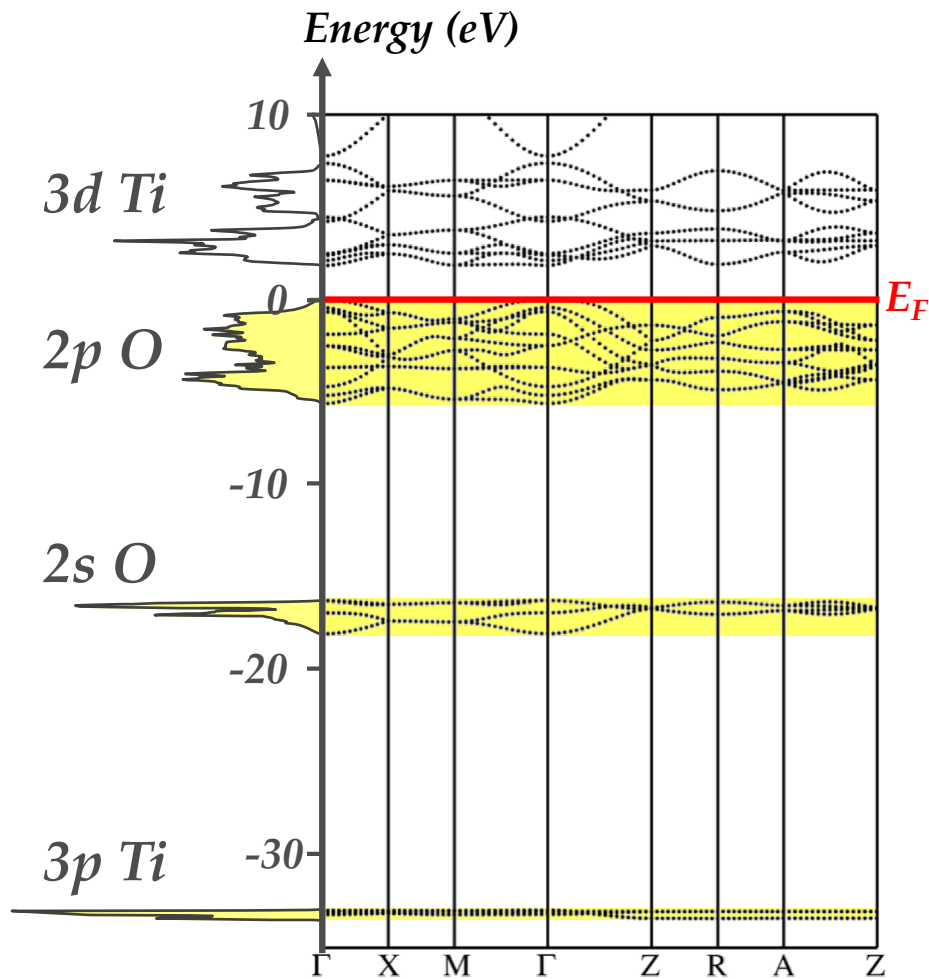
*Atomic
structure*



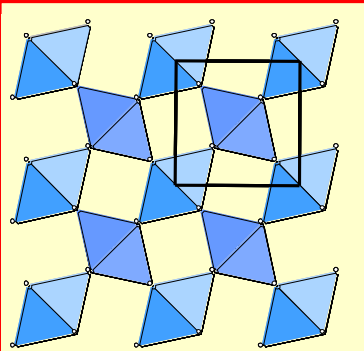
*Electronic
structure*



*Dielectric
function*



6 – ILLUSTRATIONS: TiO_2 series



Example of the rutile phase

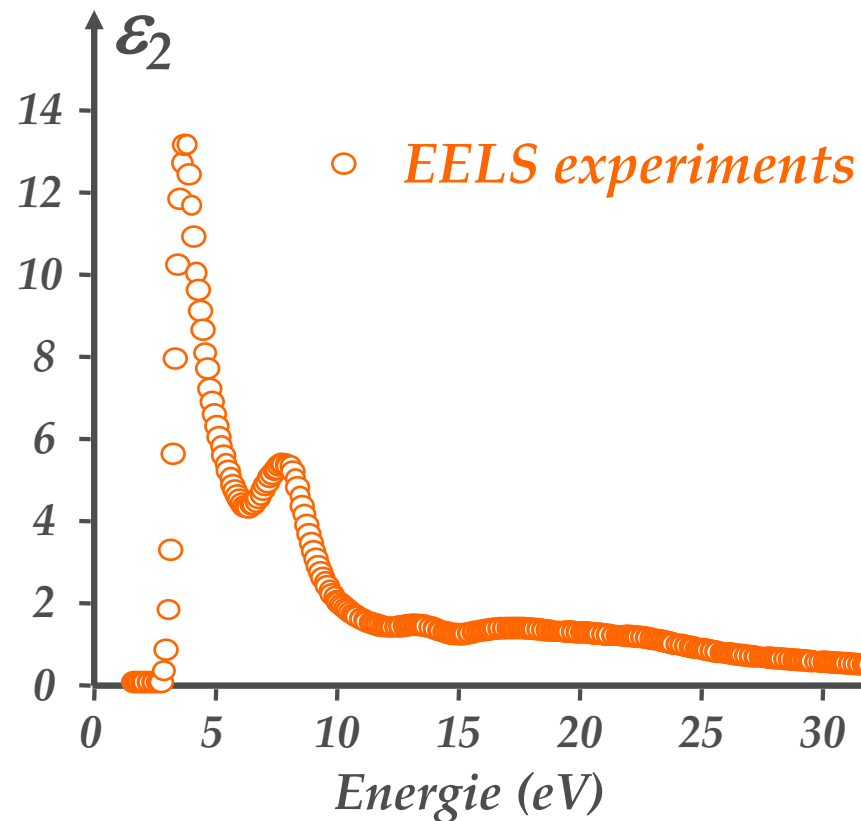
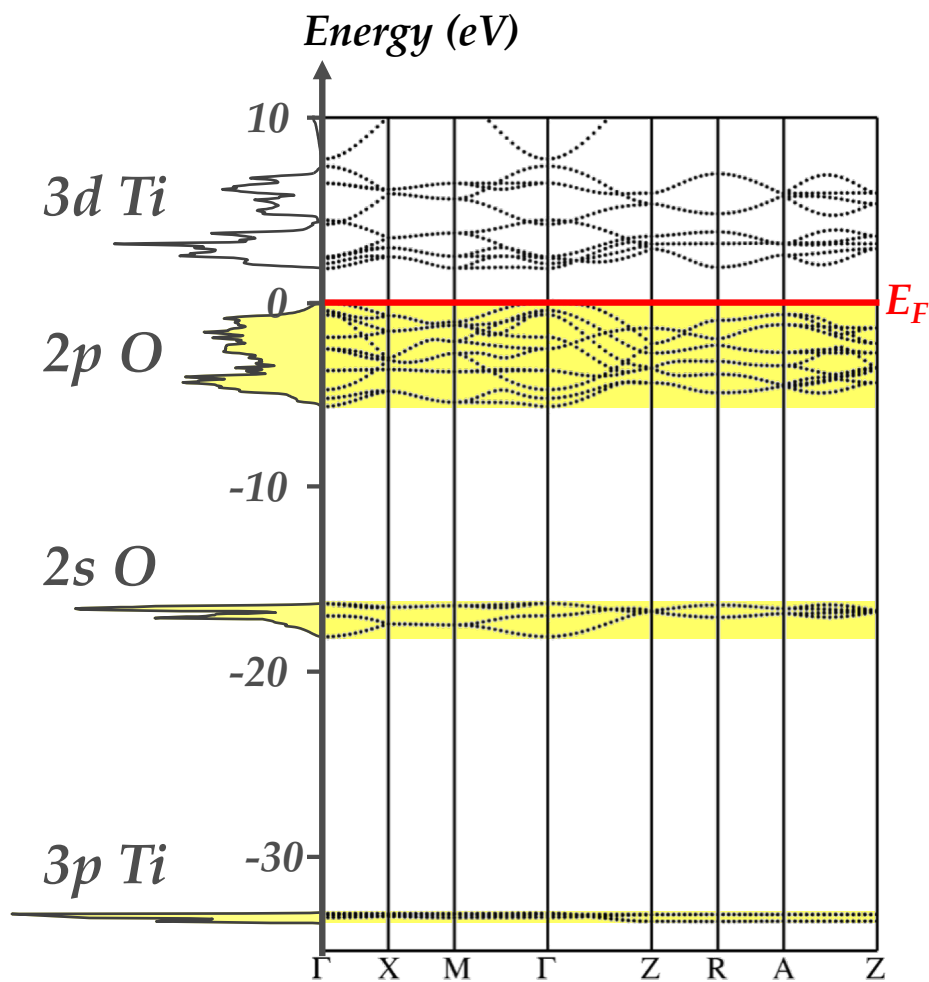
Atomic structure



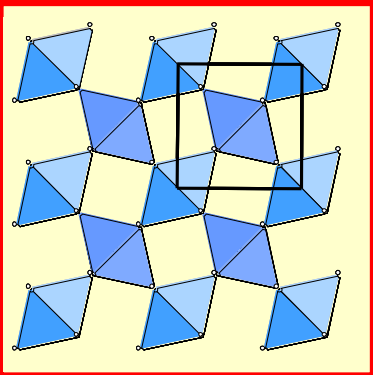
Electronic structure



Dielectric function



6 – ILLUSTRATIONS: TiO_2 series



Example of the rutile phase

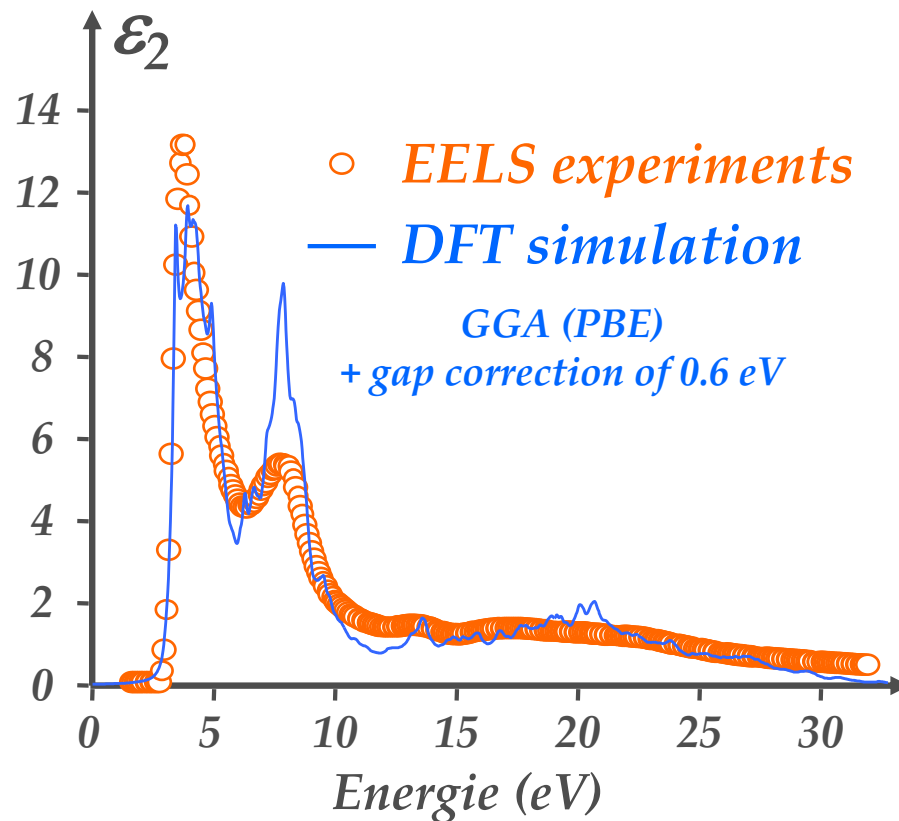
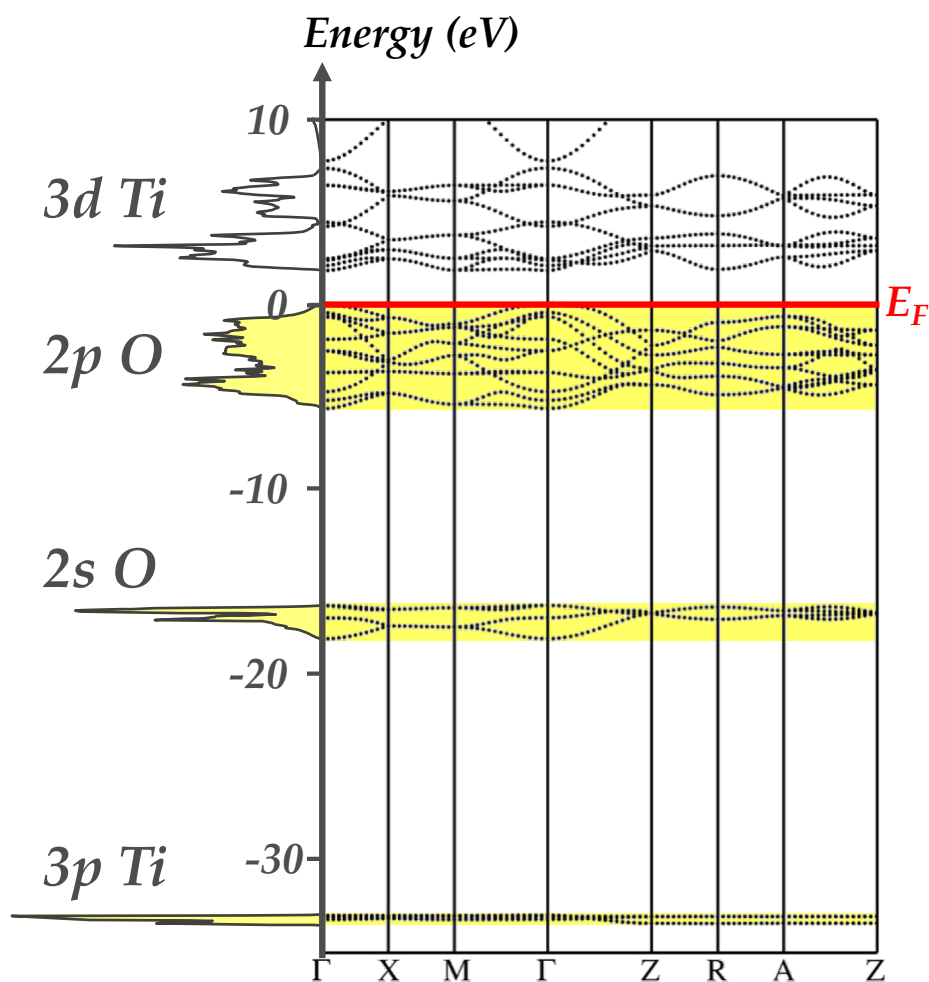
Atomic structure



Electronic structure



Dielectric function



6 – ILLUSTRATIONS: TiO_2 series

Program flow

SCF cycle → **converged potential**

x kgen → *denser k-mesh*

x lapw1 → *Kohn-Sham states for the denser k-mesh and higher E_{max}*

x lapw2 -fermi → *Fermi distribution*

x optic → *momentum matrix elements (dipolar transitions)*

x joint → *dielectric tensor components: ϵ_2*

x kram → *Kramers-Krönig transformation: $\epsilon_2 \rightarrow \epsilon_1$*

→ *Optical constants / broadening / scissors operator*

6 – ILLUSTRATIONS: TiO_2 series

TiO2-RUT.inop

2000 1 number of k-points, first k-point
-5.0 5.0 Emin, Emax *in Ry* for matrix elements
2 number of choices (columns in *outmat)
1 Re xx
3 Re zz
OFF write unsymmetrized matrix elements to file?

Choices:

1.....Re <x><x>
2.....Re <y><y>
3.....Re <z><z>
4.....Re <x><y>
5.....Re <x><z>
6.....Re <y><z>
7.....Im <x><y>
8.....Im <x><z>

6 – ILLUSTRATIONS: TiO_2 series

TiO2-RUT.injoint

| | | | | |
|--------|---------|---------|--|-------------------------------------------------|
| 1 | 261 | | | LOWER AND UPPER BANDINDEX |
| 0.0000 | 0.00100 | 10.0000 | | EMIN DE EMAX FOR ENERGYGRID IN ryd |
| | eV | | | output units eV / ryd / cm-1 |
| 4 | | | | SWITCH |
| 2 | | | | NUMBER OF COLUMNS |
| 0.1 | 0.1 | 0.3 | | BROADENING (FOR DRUDE MODEL - switch 6,7 -ONLY) |

SWITCH:

- 0...JOINTDOS FOR EACH BAND COMBINATION
- 1...JOINTDOS AS SUM OVER ALL BAND COMBINATIONS
- 2...DOS FOR EACH BAND
- 3...DOS AS SUM OVER ALL BANDS
- 4... $\text{Im}(\text{EPSILON})$
- 5... $\text{Im}(\text{EPSILON})$ for each band combination
- 6...INTRABAND contributions
- 7...INTRABAND contributions including band analysis

TiO2-RUT.inkram

| | |
|-------|-------------------------------------------------|
| 0.1 | <i>Gamma: broadening of interband spectrum</i> |
| 0.6 | <i>energy shift (scissors operator)</i> |
| 0 | <i>add intraband contributions? yes/no: 1/0</i> |
| 12.60 | <i>plasma frequencies (from joint, opt 6)</i> |
| 0.20 | <i>Gammas for Drude terms</i> |

6 – ILLUSTRATIONS: TiO_2 series

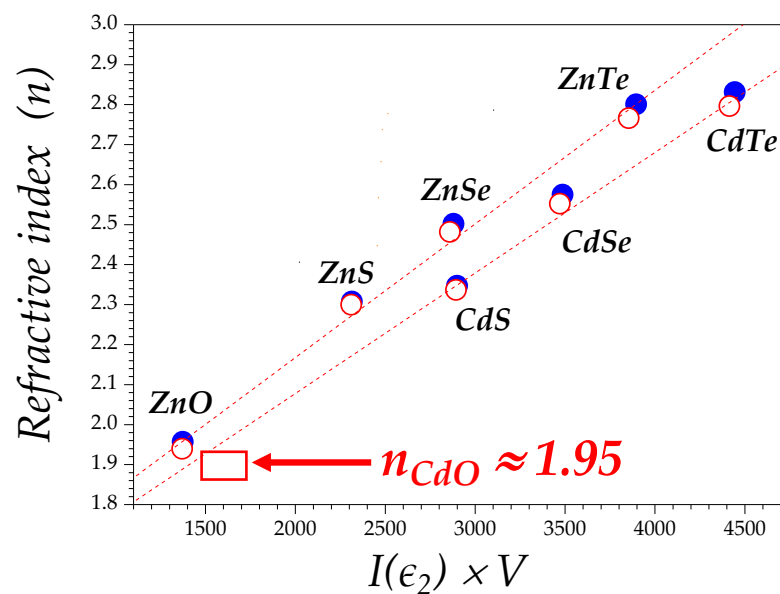
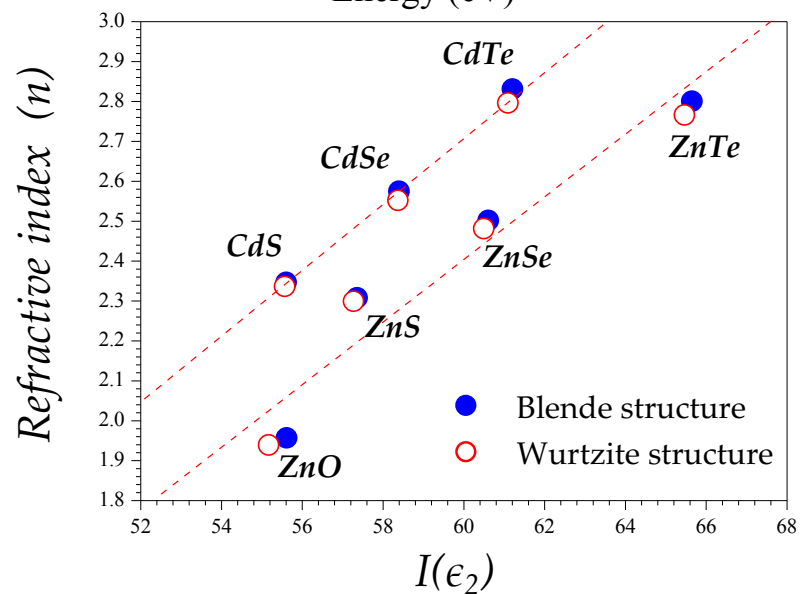
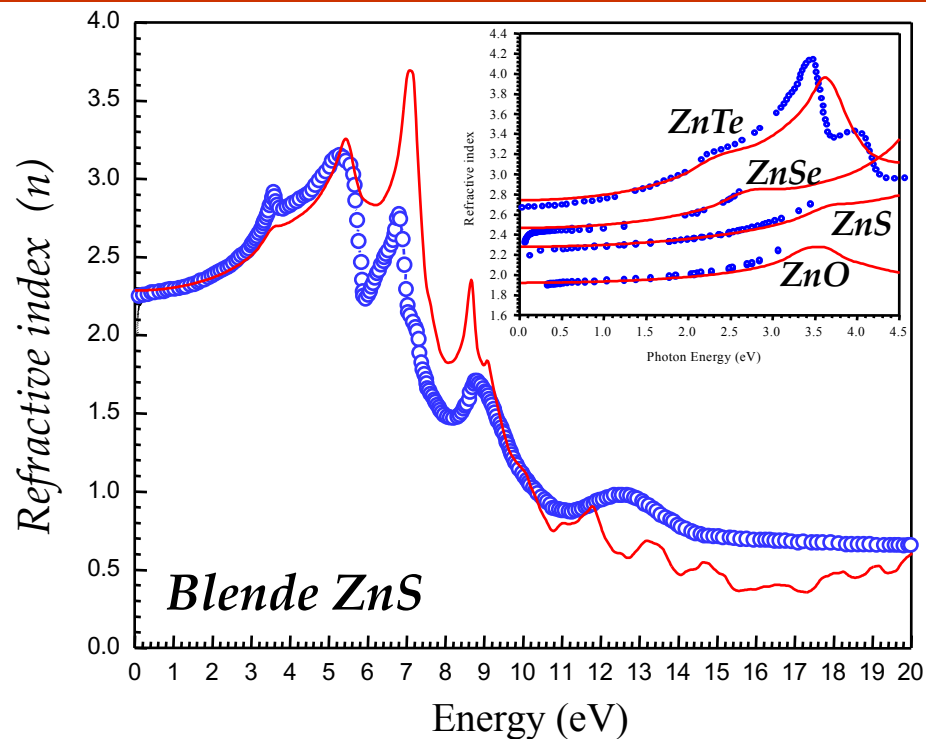
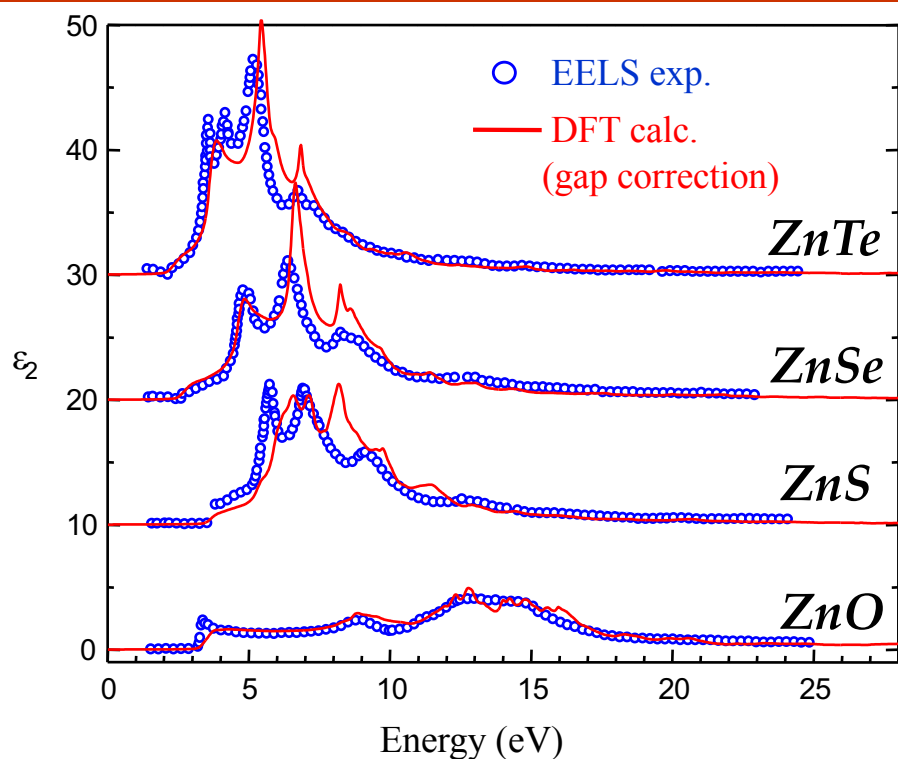
Files generated by:

x optic → TiO2-RUT.symmat
→ TiO2-RUT.mommat

x joint → TiO2-RUT.joint

x kram → TiO2-RUT.epsilon
→ TiO2-RUT.sigmak
→ TiO2-RUT.refraction
→ TiO2-RUT.absorption
→ TiO2-RUT.eloss

6 – ILLUSTRATIONS: MQ series ($M = \text{Zn, Cd}$ & $Q = \text{O, S, Se, Te}$)



7 – *Some limitations of DFT simulation of optical properties*

- **Kohn-Sham eigenstates interpreted as excited states**

 - Use of a scissors operator

- **Independent-particle approximation (no $e^- - h^+$ interaction)**

 - Use of Bethe-Salpeter Equation (BSE) – Time-dependent DFT

- **LDA/GGA are not exact**

 - Use of hybrid DFT, effective potentials

 - Use of DFT+U, LDA+DMFT, GW...

WIEN2k allows to simulate a lot of “excited states” properties

BUT

you must be aware of the limitations and approximations to properly interpret your data.